

## Exercices complémentaires : Fractions algébriques, équations et inéquations réductibles au premier degré : Solutions

1. Simplifier après avoir donné les conditions d'existence :

$$(a) \frac{x-2}{2x} + \frac{1}{x^2} - \frac{1}{2}$$

$$\underline{CE} : x \neq 0$$

$$= \frac{x(x-2) + 2 - x^2}{2x^2}$$

$$= \frac{\cancel{x^2} - 2x + 2 - \cancel{x^2}}{2x^2}$$

$$= \frac{\cancel{2}(-x+1)}{\cancel{2}x^2}$$

$$= \frac{1-x}{x^2}$$

$$(b) \frac{x-2y}{x} + \frac{2x-y}{y}$$

$$\underline{CE} : x \neq 0, y \neq 0$$

$$= \frac{y(x-2y) + x(2x-y)}{xy}$$

$$= \frac{\cancel{xy} - 2y^2 + 2x^2 - \cancel{xy}}{xy}$$

$$= \frac{2(x^2 - y^2)}{xy}$$

$$(c) \frac{1-x}{x} \ominus \frac{2x^2-1}{2x^2}$$

$$\underline{CE}: n \neq 0$$

$$= \frac{2n(1-n) \ominus (2n^2-1)}{2n^2}$$

$$= \frac{2n - 2n^2 - 2n^2 - 1}{2n^2}$$

$$= \frac{-4n^2 + 2n - 1}{2n^2}$$

$$(d) \frac{4x^2}{3x^2-3x} + \frac{5x}{x^2-1} - 1 = \frac{4x^2}{3x(x-1)} + \frac{5x}{(x-1)(x+1)} - 1$$

$$\underline{CE}: x \neq 0, x \neq 1, x \neq -1$$

$$= \frac{4x^2(x+1) + 15x^2 - 3x(x-1)(x+1)}{3x(x-1)(x+1)}$$

$$= \frac{4x^3 + 4x^2 + 15x^2 - 3x(x^2-1)}{3x(x-1)(x+1)}$$

$$= \frac{4x^3 + 19x^2 - 3x^3 + 3x}{3x(x-1)(x+1)}$$

$$= \frac{x(x^2 + 19x + 3)}{3x(x-1)(x+1)}$$

$$= \frac{x^2 + 19x + 3}{3(x-1)(x+1)}$$

$$(e) \frac{2a}{a-1} - \frac{a^2+2a}{a^2-1} = \frac{2a}{a-1} - \frac{a^2+2a}{(a-1)(a+1)}$$

CE:  $a \neq 1, a \neq -1$

$$= \frac{2a(a+1) - (a^2+2a)}{(a-1)(a+1)}$$

$$= \frac{2a^2 + \cancel{2a} - a^2 - \cancel{2a}}{D}$$

$$= \frac{a^2}{(a-1)(a+1)}$$

$$(f) \frac{1}{x-2} - \frac{1}{x+2} + \frac{2x}{x^2-4} = \frac{1}{x-2} - \frac{1}{x+2} + \frac{2x}{(x-2)(x+2)}$$

CE:  $x \neq 2, x \neq -2$

$$= \frac{x+2 - (x-2) + 2x}{(x-2)(x+2)}$$

$$= \frac{\cancel{x} + 2 - \cancel{x} + 2 + 2x}{D}$$

$$= \frac{4+2x}{D}$$

$$= \frac{2(\cancel{2+x})}{(x-2)(\cancel{x+2})}$$

$$= \frac{2}{x-2}$$

$$(g) \frac{2a}{a^2-1} - \frac{1}{a+1} - \frac{1}{a-1} = \frac{2a}{(a-1)(a+1)} - \frac{1}{a+1} - \frac{1}{a-1}$$

CE :  $a \neq 1, a \neq -1$

$$= \frac{2a - (a-1) - (a+1)}{(a-1)(a+1)}$$

$$= \frac{2a - a + 1 - a - 1}{D}$$

$$= 0$$

$$(h) \frac{3x}{x^2-xy} - \frac{2y}{xy+y^2} = \frac{3x}{x(x-y)} - \frac{2y}{y(x+y)}$$

CE :  $x \neq 0, y \neq 0, x \neq y, x \neq -y$

$$= \frac{3}{x-y} - \frac{2}{x+y}$$

$$= \frac{3(x+y) - 2(x-y)}{(x-y)(x+y)}$$

$$= \frac{3x + 3y - 2x + 2y}{D}$$

$$= \frac{x + 5y}{(x-y)(x+y)}$$

$$(i) \frac{5}{2a-4b} - \frac{a-2b}{a^2-4b^2} = \frac{5}{2(a-2b)} - \frac{a-2b}{(a-2b)(a+2b)}$$

CE  $a \neq 2b$ ,  $a \neq -2b$

$$= \frac{5}{2(a-2b)} - \frac{1}{a+2b}$$

$$= \frac{5(a+2b) - 2(a-2b)}{2(a-2b)(a+2b)}$$

$$= \frac{5a+10b-2a+4b}{2(a-2b)(a+2b)}$$

$$= \frac{3a+14b}{2(a-2b)(a+2b)}$$

$$(j) \frac{3x+3y}{x^2-y^2} + \frac{7x-7y}{x^2-2xy+y^2} = \frac{3(x+y)}{(x-y)(x+y)} + \frac{7(x-y)}{(x-y)^2}$$

CE:  $x \neq y$ ,  $x \neq -y$

$$= \frac{3}{x-y} + \frac{7}{x-y}$$

$$= \frac{10}{x-y}$$

2. Résoudre dans  $\mathbb{R}$  les équations suivantes :

F = factorisation

(a)  $x(2x - 1)(3x + 7) = 0$

$$\Leftrightarrow \begin{cases} x = 0 \\ 2x - 1 = 0 \\ 3x + 7 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = \frac{1}{2} \\ x = -\frac{7}{3} \end{cases}$$

$$S: \left\{ -\frac{7}{3}, 0, \frac{1}{2} \right\}$$

(b)  $x^2 = 64$

$$\Leftrightarrow x^2 - 64 = 0 \stackrel{F}{\Leftrightarrow} (x - 8)(x + 8) = 0$$
$$\Leftrightarrow \begin{cases} x - 8 = 0 \\ x + 8 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 8 \\ x = -8 \end{cases}$$

$$S: \{-8, 8\}$$

(c)  $9x^2 + 16 = -24x$

$$\Leftrightarrow 9x^2 + 24x + 16 = 0 \stackrel{F}{\Leftrightarrow} (3x + 4)^2 = 0$$

$$\stackrel{*}{\Leftrightarrow} 3x + 4 = 0 \Leftrightarrow x = -\frac{4}{3}$$

$$S: \left\{ -\frac{4}{3} \right\}$$

$$\stackrel{*}{\Leftrightarrow} a = 0 \Leftrightarrow a^2 = 0^2 \Leftrightarrow a^2 = 0$$

$$(d) x^3 = x$$

$$\Leftrightarrow x^3 - x = 0 \stackrel{F}{\Leftrightarrow} x(x^2 - 1) = 0 \stackrel{F}{\Leftrightarrow} x(x-1)(x+1) = 0$$

$$\Leftrightarrow \begin{cases} x = 0 \\ x - 1 = 0 \\ x + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 1 \\ x = -1 \end{cases} \quad S = \{-1, 0, 1\}$$

$$(e) (12x^4 - 3x^2) + (12x^3 - 3x) = 0$$

$$\stackrel{F}{\Leftrightarrow} 3x^2(4x^2 - 1) + 3x(4x^2 - 1) = 0$$

$$\Leftrightarrow (4x^2 - 1)(3x^2 + 3x) = 0$$

$$\stackrel{F}{\Leftrightarrow} (2x - 1)(2x + 1)(3x)(x + 1) = 0$$

$$\Leftrightarrow \begin{cases} 3x = 0 \\ 2x - 1 = 0 \\ 2x + 1 = 0 \\ x + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = \frac{1}{2} \\ x = -\frac{1}{2} \\ x = -1 \end{cases}$$

$$S = \left\{ -1, -\frac{1}{2}, 0, \frac{1}{2} \right\}$$

$$(f) 2x(x^2 - 1) = 3(x^2 - 1)$$

$$\Leftrightarrow 2x(x^2 - 1) - 3(x^2 - 1) = 0$$

$$\stackrel{F}{\Leftrightarrow} (x^2 - 1)(2x - 3) = 0$$

$$\stackrel{F}{\Leftrightarrow} (x - 1)(x + 1)(2x - 3) = 0$$

$$\Leftrightarrow \begin{cases} x - 1 = 0 \\ x + 1 = 0 \\ 2x - 3 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ x = -1 \\ x = \frac{3}{2} \end{cases} \quad S: \left\{ -1, 1, \frac{3}{2} \right\}$$

$$(g) x^2(4x - 1) + 9(1 - 4x) = 0 \Leftrightarrow x^2(4x - 1) - 9(4x - 1) = 0$$

$$\stackrel{F}{\Leftrightarrow} (4x - 1)(x^2 - 9) = 0$$

$$\stackrel{F}{\Leftrightarrow} (4x - 1)(x - 3)(x + 3) = 0$$

$$\Leftrightarrow \begin{cases} 4x - 1 = 0 \\ x - 3 = 0 \\ x + 3 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{4} \\ x = 3 \\ x = -3 \end{cases} \quad S: \left\{ -3, \frac{1}{4}, 3 \right\}$$

$$(h) (3x - 1)(x + 2) = x(x + 2)$$

$$\Leftrightarrow (3x - 1)(x + 2) - x(x + 2) = 0$$

$$\stackrel{F}{\Leftrightarrow} (x + 2)[(3x - 1) - x] = 0$$

$$\Leftrightarrow (x + 2)(2x - 1) = 0$$

$$\Leftrightarrow \begin{cases} x + 2 = 0 \\ 2x - 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -2 \\ x = \frac{1}{2} \end{cases}$$

$$S: \left\{ -2, \frac{1}{2} \right\}$$



$$(i) (x-1)(3x-2) = 4x(2-3x)$$

$$\Leftrightarrow (x-1)(3x-2) - 4x(2-3x) = 0$$

$$\Leftrightarrow (x-1)(3x-2) + 4x(3x-2) = 0$$

$$\Leftrightarrow (3x-2)[(x-1)+4x] = 0$$

$$\Leftrightarrow (3x-2)(5x-1) = 0$$

$$\Leftrightarrow \begin{cases} 3x-2=0 \\ 5x-1=0 \end{cases} \Leftrightarrow \begin{cases} x = \frac{2}{3} \\ x = \frac{1}{5} \end{cases} \quad S: \left\{ \frac{1}{5}, \frac{2}{3} \right\}$$

$$(j) 2x^3 + x^2 + 2x = -1$$

$$\Leftrightarrow (2x^3 + x^2) + (2x + 1) = 0$$

$$\Leftrightarrow x^2(2x+1) + (2x+1) = 0$$

$$\Leftrightarrow (2x+1)(x^2+1) = 0$$

$$\Leftrightarrow \begin{cases} 2x+1=0 \\ x^2+1=0 \end{cases} \Leftrightarrow \begin{cases} x = -\frac{1}{2} \\ x^2+1=0 \rightarrow \text{imp} \end{cases}$$

$$S: \left\{ -\frac{1}{2} \right\}$$

3. Résoudre les équations fractionnaires suivantes :

$$(a) \frac{2}{x-3} - \frac{1}{x} = \frac{1}{x^2-3x} \Leftrightarrow \frac{2}{x-3} - \frac{1}{x} = \frac{1}{x(x-3)}$$

CE:  $x \neq 0, x \neq 3$

$$\Leftrightarrow \frac{2x - (x-3)}{x(x-3)} = \frac{1}{x(x-3)}$$

$$\Leftrightarrow x + 3 = 1 \Leftrightarrow x = -2$$

$$S: \{-2\}$$

$$(b) \frac{3}{x-1} - \frac{1}{x+2} = \frac{-5}{1-x}$$

CE:  $x \neq 1, x \neq -2$

$$\Leftrightarrow \frac{3(x+2) - (x-1)}{(x-1)(x+2)} = \frac{5(x+2)}{(x-1)(x+2)} \quad (*)$$

$$\Leftrightarrow 3x + 6 - x + 1 = 5x + 10$$

$$\Leftrightarrow -3x = 3$$

$$\Leftrightarrow x = -1$$

$$S: \{-1\}$$

$(*)$  car  $\frac{-5}{1-x} = \frac{5}{x-1}$

$$(c) 1 - \frac{x}{x-2} + \frac{1}{x+2} = 0$$

$$\underline{\text{CE}} : x \neq 2, x \neq -2$$

$$\Leftrightarrow \frac{(x-2)(x+2) - x(x+2) + (x-2)}{(x-2)(x+2)} = 0$$

$$\Leftrightarrow x^2 - 4 - x^2 - 2x + x - 2 = 0$$

$$\Leftrightarrow -x - 6 = 0$$

$$\Leftrightarrow x = -6$$

$$S: \{-6\}$$

$$(d) \frac{-1}{x+3} - \frac{2}{x-1} = \frac{1}{(x+3)(x-1)}$$

$$\underline{\text{CE}} : x \neq 1, x \neq -3$$

$$\Leftrightarrow \frac{-(x-1) - 2(x+3)}{(x+3)(x-1)} = \frac{1}{(x+3)(x-1)}$$

$$\Leftrightarrow -x + 1 - 2x - 6 = 1$$

$$\Leftrightarrow -3x = 6$$

$$\Leftrightarrow x = -2$$

$$S: \{-2\}$$

$$(e) \frac{1}{x-1} + \frac{2}{x+1} = \frac{1}{x^2-2x+1} \Leftrightarrow \frac{1}{(x-1)} + \frac{2}{(x+1)} = \frac{1}{(x-1)^2}$$

$$\underline{CE}: x \neq 1, x \neq -1$$

$$\Leftrightarrow \frac{(x-1)(x+1) + 2(x-1)^2}{(x-1)^2(x+1)} = \frac{(x+1)}{(x-1)^2(x+1)}$$

$$\Leftrightarrow x^2 - 1 + 2(x^2 - 2x + 1) = x + 1$$

$$\Leftrightarrow 3x^2 - 5x = 0$$

$$\Leftrightarrow x(3x - 5) = 0$$

$$\Leftrightarrow \begin{cases} x = 0 \\ 3x - 5 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = \frac{5}{3} \end{cases}$$

$$S: \left\{ 0, \frac{5}{3} \right\}$$

$$(f) \frac{2}{(3x+2)(7x-1)} = \frac{1}{3x+2} + \frac{3}{7x-1} \quad \underline{CE} \quad x \neq -\frac{2}{3}, x \neq \frac{1}{7}$$

$$\Leftrightarrow \frac{2}{(3x+2)(7x-1)} = \frac{7x-1 + 3(3x+2)}{(3x+2)(7x-1)}$$

$$\Leftrightarrow 2 = 7x - 1 + 9x + 6$$

$$\Leftrightarrow -3 = 16x$$

$$\Leftrightarrow x = -\frac{3}{16}$$

$$S: \left\{ -\frac{3}{16} \right\}$$

$$(g) \frac{2}{x-3} - \frac{1}{x+1} = \frac{4}{(x-3)(x+1)} - \frac{2}{x-3} \quad \underline{CE}: x \neq -1, x \neq 3$$

$$\Leftrightarrow \frac{2(x+1) - (x-3)}{\cancel{(x+1)}\cancel{(x-3)}} = \frac{4 - 2(x+1)}{\cancel{(x+1)}\cancel{(x-3)}}$$

$$\Leftrightarrow 2x + 2 - x + 3 = 4 - 2x - 2$$

$$\Leftrightarrow 3x = -3$$

$$\Leftrightarrow x = -1 \quad (\text{A.R. per CE})$$

$$S: \emptyset$$

$$(h) \frac{3}{x-2} - \frac{1}{2x+5} + \frac{2}{2x^2+x-10} = 0$$

Denominator

$$\Leftrightarrow \frac{3}{x-2} - \frac{1}{2x+5} + \frac{2}{(2x+5)(x-2)} = 0$$

$$\underline{CE}: x \neq 2, x \neq -\frac{5}{2}$$

$$\Leftrightarrow \frac{3(2x+5) - (x-2) + 2}{\cancel{(x-2)}\cancel{(2x+5)}} = 0$$

$$\Leftrightarrow 6x + 15 - x + 2 + 2 = 0$$

$$\Leftrightarrow 5x + 19 = 0$$

$$\Leftrightarrow x = -\frac{19}{5}$$

$$S: \left\{ -\frac{19}{5} \right\}$$

$$(i) \frac{1}{4x-3} - \frac{2}{2x+1} = \frac{3}{(4x-3)(2x+1)} \quad \underline{CE} : x \neq \frac{3}{4}, x \neq -\frac{1}{2}$$

$$\Leftrightarrow \frac{(2x+1) - 2(4x-3)}{(4x-3)(2x+1)} = \frac{3}{(4x-3)(2x+1)}$$

$$\Leftrightarrow 2x+1 - 8x+6 = 3$$

$$\Leftrightarrow -6x = -4$$

$$\Leftrightarrow x = \frac{2}{3}$$

$$S : \left\{ \frac{2}{3} \right\}$$

$$(ii) \frac{2}{3x-1} - \frac{-1}{2x-1} - \frac{1}{(2x-1)(3x-1)} = 0 \quad \underline{CE} : x \neq \frac{1}{3}, x \neq \frac{1}{2}$$

$$\Leftrightarrow \frac{2(2x-1) - (-1)(3x-1) - 1}{(3x-1)(2x-1)} = 0$$

$$\Leftrightarrow 4x - 2 + 3x - 1 - 1 = 0$$

$$\Leftrightarrow 7x = 4$$

$$\Leftrightarrow x = \frac{4}{7}$$

$$S : \left\{ \frac{4}{7} \right\}$$

4. Résoudre les équations et inéquations suivantes :

$$(a) \frac{7x^2 - 3x + 1}{3x - 8} = 2x - 3$$

$$\underline{CE} : x \neq \frac{8}{3}$$

$$\Leftrightarrow \frac{7x^2 - 3x + 1}{\cancel{3x - 8}} = \frac{(2x - 3)(\cancel{3x - 8})}{\cancel{3x - 8}}$$

$$\Leftrightarrow 7x^2 - 3x + 1 = 6x^2 - 16x - 9x + 24$$

$$\Leftrightarrow x^2 + 22x - 23 = 0$$

Momem

$$\Leftrightarrow (x - 1)(x + 23) = 0$$

$$\Leftrightarrow \begin{cases} x - 1 = 0 \\ x + 23 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ x = -23 \end{cases}$$

$$S : \{ -23, 1 \}$$

$$(b) \frac{x^4 - 7x^3 - 8x^2}{5x^3 + 2x - 7} = 0$$

$$\begin{aligned} \text{CE} & 5x^3 + 2x - 7 \neq 0 \\ \text{Hörner} & \\ \Rightarrow & (x-1)(5x^2 + 5x + 7) \neq 0 \\ \Rightarrow & x \neq 1 \end{aligned}$$

$$\Leftrightarrow x^4 - 7x^3 - 8x^2 = 0$$

$$\Leftrightarrow x^2(x^2 - 7x - 8) = 0$$

Hörner

$$\Leftrightarrow x^2(x+1)(x-8) = 0$$

$$\Leftrightarrow \begin{cases} x^2 = 0 \\ x+1 = 0 \\ x-8 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = -1 \\ x = 8 \end{cases}$$

$$S: \{-1, 0, 8\}$$



$$(c) \frac{(x-1)^3}{x^2} = x-3$$

$$\underline{CE} : x \neq 0$$

$$\Leftrightarrow \frac{(x-1)^3}{\cancel{x^2}} = \frac{x^2(x-3)}{\cancel{x^2}}$$

$$\Leftrightarrow x^3 - 3x^2 + 3x - 1 = x^3 - 3x^2$$

$$\Leftrightarrow 3x - 1 = 0$$

$$\Leftrightarrow x = \frac{1}{3}$$

$$S. \left\{ \frac{1}{3} \right\}$$

$$(d) x+1 = \frac{1-2x}{x+1}$$

CE

$$x \neq -1$$

$$\Leftrightarrow \frac{(x+1)^2}{\cancel{x+1}} = \frac{1-2x}{\cancel{x+1}}$$

$$\Leftrightarrow x^2 + 2x + 1 = 1 - 2x$$

$$\Leftrightarrow x^2 + 4x = 0$$

$$\Leftrightarrow x(x+4) = 0$$

$$\Leftrightarrow \begin{cases} x = 0 \\ x + 4 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = -4 \end{cases}$$

$$S: \{-4, 0\}$$

$$(e) \frac{2x-1}{x-2} - \frac{4x-1}{x+2} = 1$$

$$\underline{CE} \quad x \neq 2, \quad x \neq -2$$

$$\Leftrightarrow \frac{(2x-1)(x+2) - (4x-1)(x-2)}{(x-2)(x+2)} = \frac{(x+2)(x-2)}{(x+2)(x-2)}$$

$$\Leftrightarrow 2x^2 + 4x - x - 2 - (4x^2 - 8x - x + 2) = x^2 - 4$$

$$\Leftrightarrow x^2 + 3x + \cancel{2} - 4x^2 + 9x - \cancel{2} = 0$$

$$\Leftrightarrow -3x^2 + 12x = 0$$

$$\Leftrightarrow -3x(x-4) = 0$$

$$\Leftrightarrow \begin{cases} -3x = 0 \\ x - 4 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 4 \end{cases}$$

$$S = \{0, 4\}$$

$$(f) \frac{-12}{x-4} \geq x+3$$

$$(CE: x-4 \neq 0 \Rightarrow x \neq 4)$$

$$\Leftrightarrow -\frac{12}{x-4} - (x+3) \geq 0$$

$$\Leftrightarrow \frac{-12 - (x-4)(x+3)}{x-4} \geq 0$$

$$\Leftrightarrow \frac{-12 - (x^2 - 4x + 3x - 12)}{x-4} \geq 0$$

$$\Leftrightarrow \frac{-12 - x^2 + x + 12}{x-4} \geq 0$$

$$\Leftrightarrow \frac{-x^2 + x}{x-4} \geq 0$$

$$\Leftrightarrow \frac{x(-x+1)}{x-4} \geq 0$$

Zeits  $\underline{N}$ :  $x=0$  ;  $x=1$

$\underline{D}$ :  $x=4$

	$x$		0	1	4	
$N$	$x$	-	0	+	+	+
	$-x+1$	+	+	0	-	-
$D$	$x-4$	-	-	-	0	+
$I_n$		+	0	-	0	+

$x$  ————  $\bullet$  ————  $\bullet$  ————  $x$

$$S: -\infty, 0] \cup [1, 4[$$

$$(g) x + \frac{1}{x} > 2$$

$$\Leftrightarrow x + \frac{1}{x} - 2 > 0 \Leftrightarrow \frac{x^2 + 1 - 2x}{x} > 0$$

$$\Leftrightarrow \frac{(x-1)^2}{x} > 0$$

zeros: N:  $x = 1$

D:  $x = 0$

$x$		0		1	
$(x-1)^2$		+		+	+
$x$		-	0	+	+
$I_m$		-	<del>+</del>	+	+
			x		x

$$S: ]0, 1[ \cup ]1, +\infty$$

ou  $S: \mathbb{R}_0^+ \setminus \{1\}$

$$(h) \frac{2x-3}{x-1} \leq x-1$$

$$\Leftrightarrow \frac{2x-3}{x-1} - x - 1 \leq 0$$

$$\Leftrightarrow \frac{2x-3 - (x-1)^2}{x-1} \leq 0$$

$$\Leftrightarrow \frac{2x-3 - (x^2 - 2x + 1)}{(x-1)} \leq 0$$

$$\Leftrightarrow \frac{-x^2 + 4x - 4}{x-1} \leq 0$$

$$\Leftrightarrow \frac{-(x^2 - 4x + 4)}{x-1} \leq 0$$

$$\Leftrightarrow \frac{-(x-2)^2}{x-1} \leq 0$$

Zeig: N:  $x=2$

D:  $x=1$

$x$		1		2	
$-(x-2)^2$		-		0	-
$x-1$		-	0	+	+
$I_n$		+	<del>+</del>	-	0

$x$   $x$

S:  $]1, +\infty$

$$(i) 1 + \frac{1}{x+2} \leq \frac{x}{x-2}$$

$$\Leftrightarrow 1 + \frac{1}{x+2} - \frac{x}{x-2} \leq 0$$

$$\Leftrightarrow \frac{(x+2)(x-2) + (x-2) - x(x+2)}{(x-2)(x+2)} \leq 0$$

$$\Leftrightarrow \frac{\cancel{x^2} - 4 + x - 2 - \cancel{x^2} - 2x}{D} \leq 0$$

$$\Leftrightarrow \frac{-x - 6}{(x-2)(x+2)} \leq 0$$

Zeilen: N :  $x = -6$

D :  $x = -2, x = 2$

$x$		$-6$	$-2$	$2$		
$-x - 6$	+	0	-	-	-	
$x - 2$	-		-	-	0	+
$x + 2$	-		-	0	+	+
$I_n$	+	0	-	<del>+</del>	+	<del>-</del>

$$S: [-6, -2[ \cup ]2, +\infty$$

$$(j) \frac{2}{(3x+2)(7x-1)} > \frac{1}{3x+2} + \frac{3}{7x-1}$$

$$\Leftrightarrow \frac{2}{(3x+2)(7x-1)} - \frac{1}{3x+2} - \frac{3}{7x-1} > 0$$

$$\Leftrightarrow \frac{2 - (7x-1) - 3(3x+2)}{(3x+2)(7x-1)} > 0$$

$$\Leftrightarrow \frac{-16x - 3}{(3x+2)(7x-1)} > 0$$

zeros : N :  $x = -\frac{3}{16}$

D :  $x = -\frac{2}{3}, x = \frac{1}{7}$

$x$		$-\frac{2}{3}$	$-\frac{3}{16}$	$\frac{1}{7}$	
$-16x-3$	+	-	+	0	-
$3x+2$	-	0	+	+	+
$7x-1$	-	-	-	0	+
$I_n$	+	<del>-</del>	-	0	+
		X	X	X	X

S :  $-\infty, -\frac{2}{3} [ \cup ] -\frac{3}{16}, \frac{1}{7} [$



$$(k) \frac{3}{x-2} + \frac{2}{2x^2+x-10} \geq \frac{1}{2x+5}$$

$$\hookrightarrow \text{Nenner: } 2x^2+x-10 = (x-2)(2x+5)$$

$$\Leftrightarrow \frac{3}{x-2} + \frac{2}{(x-2)(2x+5)} \ominus \frac{1}{2x+5} \geq 0$$

$$\Leftrightarrow \frac{3(2x+5) + 2 - (x-2)}{(x-2)(2x+5)} \geq 0$$

$$\Leftrightarrow \frac{5x+19}{(x-2)(2x+5)} \geq 0$$

Zeiler: N:  $x = -\frac{19}{5}$

D:  $x = 2, x = -\frac{5}{2}$

$x$		$-\frac{19}{5}$	$-\frac{5}{2}$	$2$	
$5x+19$	-	0	+	+	+
$x-2$	-	-	-	0	+
$2x+5$	-	-	0	+	+
$I_n$	-	0	<del>+</del>	<del>-</del>	+

$$S: \left[-\frac{19}{5}, -\frac{5}{2}\right] \cup [2, +\infty)$$

$$(1) \frac{x-1}{x} + \frac{x}{x-2} < \frac{4}{x^2-2x}$$

$$\Leftrightarrow \frac{x-1}{x} + \frac{x}{x-2} - \frac{4}{x(x-2)} < 0$$

$$\Leftrightarrow \frac{(x-1)(x-2) + x^2 - 4}{x(x-2)} < 0$$

$$\Leftrightarrow \frac{x^2 - 3x + 2 + x^2 - 4}{x(x-2)} < 0$$

$$\Leftrightarrow \frac{2x^2 - 3x - 2}{x(x-2)} < 0$$

zeros  $N$ :  $x = 2, x = -\frac{1}{2}$  (Horna)  
 $(x-2)(2x+1)$   
 $D$ :  $x = 0, x = 2$

$x$		$-\frac{1}{2}$	$0$	$2$	
$x-2$	-	-	-	-	0 +
$2x+1$	-	0	+	+	+
$x$	-	-	0	+	+
$x-2$	-	-	-	-	0 +
$I_n$	+	0	-	+	? +

x ————— x

$$S: ]-\frac{1}{2}, 0[$$

$$(m) \frac{x+1}{2x+4} + \frac{1}{x+1} < \frac{1}{x^2+3x+2}$$

↳ Horner  $(x+2)(x+1)$

$$\Leftrightarrow \frac{x+1}{2(x+2)} + \frac{1}{x+1} - \frac{1}{(x+2)(x+1)} < 0$$

$$\Leftrightarrow \frac{(x+1)(x+1) + 2(x+2) - 2}{2(x+2)(x+1)} < 0$$

$$\Leftrightarrow \frac{x^2 + 4x + 1 + 2x + 4 - 2}{D} < 0$$

$$\Leftrightarrow \frac{x^2 + 4x + 3}{x(x+2)(x+1)} < 0$$

Zeilen:  $N$ :  $x = -3, x = -1$  (Horner)  
 $(x+3)/(x+1)$

$D$ :  $x = -2, x = -1$

$x$		-3	-2	-1	
$x+3$	-	0	+	+	+
$x+1$	-	-	-	0	+
$x+2$	-	-	0	+	+
$x+1$	-	-	-	0	+
$I_n$	+	0	-	+	?

x ————— x

$S$ :  $] -3, -2[$

$$(n) \frac{2}{3x-1} - \frac{-1}{2x-1} > \frac{1}{(3x-1)(2x-1)}$$

$$\Leftrightarrow \frac{2}{3x-1} + \frac{+1}{2x-1} - \frac{1}{(3x-1)(2x-1)} > 0$$

$$\Leftrightarrow \frac{2(2x-1) + (3x-1) - 1}{(3x-1)(2x-1)} > 0$$

$$\Leftrightarrow \frac{7x-4}{(3x-1)(2x-1)} > 0$$

zeros : N :  $x = \frac{4}{7}$

D) :  $x = \frac{1}{2}, x = \frac{1}{3}$

$x$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{4}{7}$	
$7x-4$	-	-	- 0	+
$3x-1$	- 0	+	+	+
$2x-1$	-	- 0	+	+
$I_n$	-	<del>+</del>	<del>-</del> 0	+

x-----x
x-----x

S:  $] \frac{1}{3}, \frac{1}{2} [ \cup ] \frac{4}{7}, +\infty$

$$(o) \frac{-1}{x+3} - \frac{2}{x-1} \geq \frac{1}{x^2+2x-3}$$

↳ Momen:  $(x+3)(x-1)$

$$\Leftrightarrow \frac{-1}{x+3} - \frac{2}{x-1} - \frac{1}{(x+3)(x-1)} \geq 0$$

$$\Leftrightarrow \frac{-(x-1) - 2(x+3) - 1}{(x+3)(x-1)} \geq 0$$

$$\Leftrightarrow \frac{-3x-6}{(x+3)(x-1)} \geq 0$$

zeros N  $x = -2$

D  $x = 1, x = -3$

$x$		-3		-2		1	
$-3x-6$		+		+	0	-	-
$x+3$		-	0	+		+	+
$x-1$		-		-		-	0
$I_n$		+	<del>+</del>	-	0	+	<del>+</del>
		x	x		0	x	

$$S: -\infty, -3[ \cup [-2, 1[$$