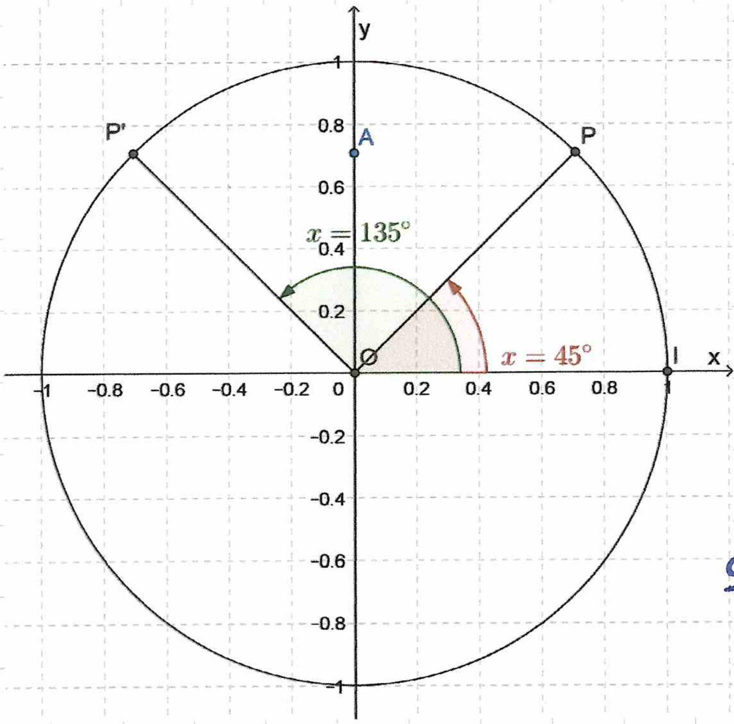
Equations trigonométriques : Solutions

1. Résoudre graphiquement les équations suivantes :

   (a) $\sin x = \frac{\sqrt{2}}{2}$



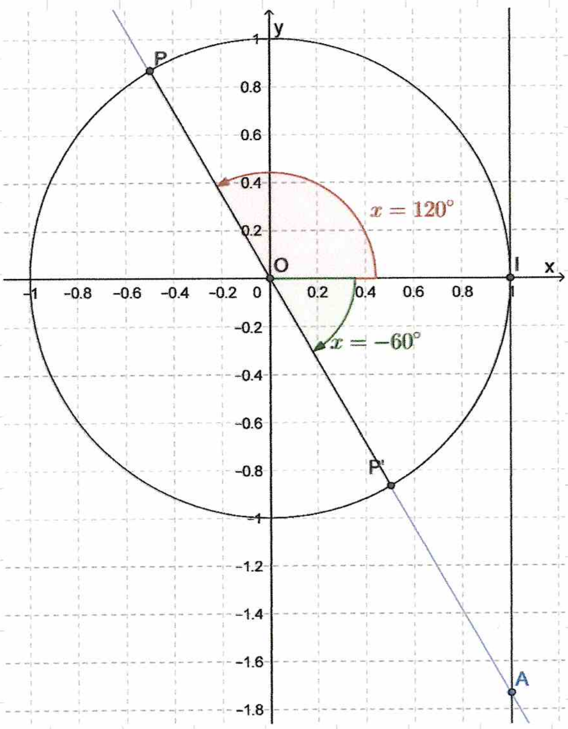
$$\begin{cases} x = 45° \\ x = 135° \end{cases}$$ lect. graph

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{4} + 2k\pi \ (*) \\ x = \frac{3\pi}{4} + 2k\pi \end{cases}$$

(*) $180° = \pi$ rad
(règle de 3)

$S_g : \left\{ \frac{\pi}{4} + 2k\pi , \frac{3\pi}{4} + 2k\pi \right\} \{ k \in \mathbb{Z} \}$

   (b) $\tan x = -\sqrt{3}$



$$\begin{cases} x = 120° \\ x = -60° \end{cases}$$

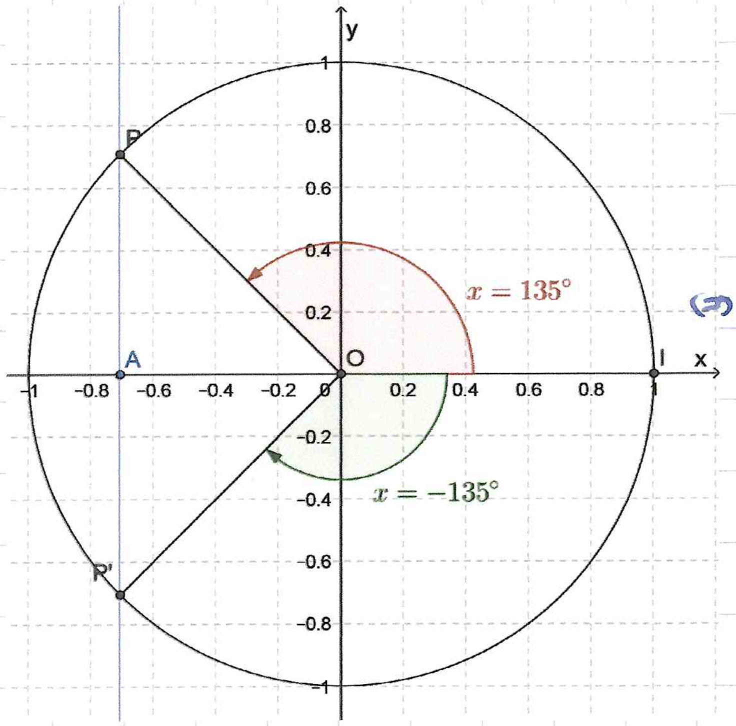
$$\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ x = -\frac{\pi}{3} + 2k\pi \end{cases}$$

ou $$\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ x = \frac{5\pi}{3} + 2k\pi \end{cases}$$

ou $x = \frac{2\pi}{3} + k\pi$

$S_g : \left\{ \frac{2\pi}{3} + k\pi \right\} k \in \mathbb{Z} \}$

(c) $1+\sqrt{2}\cos x = 0$

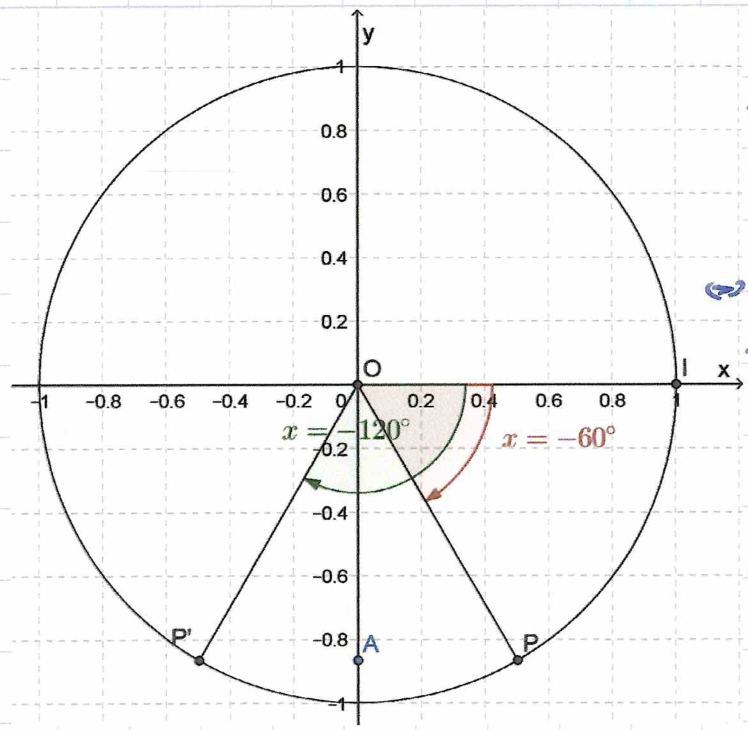


$$\begin{cases} x = 135° \\ x = -135° \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{3\pi}{4} + 2k\pi \\ x = -\frac{3\pi}{4} + 2k\pi \end{cases}$$

$$S_g : \left\{ \pm \frac{3\pi}{4} + 2k\pi \mid k \in \mathbb{Z} \right\}$$

(d) $2\sqrt{3}\sin x + 3 = 0$



$$\begin{cases} x = -60° \\ x = -120° \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ x = -\frac{2\pi}{3} + 2k\pi \end{cases}$$

ou

$$\begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ x = \frac{4\pi}{3} + 2k\pi \end{cases}$$

$$S_g : \left\{ -\frac{\pi}{3} + 2k\pi , \frac{4\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\}$$

2. Résoudre les équations suivantes

(a) $\sin x = \dfrac{1}{4}$

$\arcsin \dfrac{1}{4} \simeq 0{,}253$

$$\begin{cases} x \simeq 0{,}253 + 2k\pi \\ x = 2{,}889 + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$$

$$S_g = \{0{,}253 + 2k\pi \,;\, 2{,}889 + 2k\pi \mid k \in \mathbb{Z}\}$$

(b) $\dfrac{\cot x}{5} - \dfrac{\cot x}{3} = \dfrac{1}{4}$

$\Leftrightarrow \dfrac{-2\cot x}{15} = \dfrac{1}{4} \Leftrightarrow \cot x = -\dfrac{15}{8} \Leftrightarrow \tan x = -\dfrac{8}{15}$

$\arctan\left(-\dfrac{8}{15}\right) \simeq -0{,}49$

$x \simeq -0{,}49 + k\pi \quad (k \in \mathbb{Z})$

$$S_g = \{-0{,}49 + k\pi \mid k \in \mathbb{Z}\}$$

(c) $\sqrt{12}\cos x = \cos x - 2$

$\Leftrightarrow (\sqrt{12} - 1)\cos x = -2 \Leftrightarrow \cos x = \dfrac{-2}{\sqrt{12} - 1}$

$\arccos\left(\dfrac{-2}{\sqrt{12} - 1}\right) \simeq 2{,}518$

$$\begin{cases} x \simeq 2{,}518 + 2k\pi \\ x \simeq -2{,}518 + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$$

$$S_g = \{\pm 2{,}518 + 2k\pi \mid k \in \mathbb{Z}\}$$

3. Résoudre les équations suivantes :

(a) $\sin 3x = \sin x$

$$\begin{cases} 3x = x + 2k\pi \\ 3x = \pi - x + 2k\pi \end{cases} \Leftrightarrow \begin{cases} 2x = 2k\pi \\ 4x = \pi + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = k\pi \\ x = \frac{\pi}{4} + k\frac{\pi}{2} \end{cases}$$

$S_g : \{ k\pi, \frac{\pi}{4} + k\frac{\pi}{2} \mid k \in \mathbb{Z} \}$

(b) $\cos 2x = \cos\left(x - \frac{\pi}{3}\right)$

$$\begin{cases} 2x = x - \frac{\pi}{3} + 2k\pi \\ 2x = -\left(x - \frac{\pi}{3}\right) + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ 3x = \frac{\pi}{3} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ x = \frac{\pi}{9} + \frac{2k\pi}{3} \end{cases}$$

$S_g = \{ -\frac{\pi}{3} + 2k\pi, \frac{\pi}{9} + \frac{2k\pi}{3} \mid k \in \mathbb{Z} \}$

(c) $\cos\left(5x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$

$$\begin{cases} 5x + \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi \\ 5x + \frac{\pi}{3} = -\frac{\pi}{4} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 5x = -\frac{\pi}{12} + 2k\pi \\ 5x = -\frac{7\pi}{12} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{60} + \frac{2k\pi}{5} \\ x = -\frac{7\pi}{60} + \frac{2k\pi}{5} \end{cases}$$

$S_g : \{ -\frac{\pi}{60} + 2k\frac{\pi}{5}, -\frac{7\pi}{60} + \frac{2k\pi}{5} \mid k \in \mathbb{Z} \}$

(d) $\sin 3x = \sin\left(2x - \frac{\pi}{4}\right)$

$$\begin{cases} 3x = 2x - \frac{\pi}{4} + 2k\pi \\ 3x = \pi - \left(2x - \frac{\pi}{4}\right) + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + 2k\pi \\ 5x = \frac{5\pi}{4} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + 2k\pi \\ x = \frac{\pi}{4} + \frac{2k\pi}{5} \end{cases}$$

$$S_g : \left\{ -\frac{\pi}{4} + 2k\pi , \frac{\pi}{4} + \frac{2k\pi}{5} \mid k \in \mathbb{Z} \right\}$$

(e) $\tan\left(-x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{3}$

$$-x + \frac{\pi}{4} = -\frac{\pi}{6} + k\pi$$

$$\Leftrightarrow -x = -\frac{5\pi}{12} + k\pi \Leftrightarrow x = \frac{5\pi}{12} + k\pi$$

$$S_g : \left\{ \frac{5\pi}{12} + k\pi \mid k \in \mathbb{Z} \right\}$$

Après avoir résolu ces équations, exprimer explicitement les solutions comprises dans l'intervalle ~~$]-\pi, \pi]$~~ $[0, 2\pi[$

a) $\begin{cases} x = k\pi \\ x = \frac{\pi}{4} + k\frac{\pi}{2} \end{cases}$

(*1) $k = 0, 1$ $\quad x = 0, \pi$

(*2) $k = 0, 1, 2, 3$ $\quad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(*1) Pour trouver tts les sol, il faut donner 2 valeurs à k (2π)

(*2) Pour trouver tts les solutions, il faut donner 4 valeurs à k ($4\frac{\pi}{2} = 2\pi$)

$$S_p : \left\{ 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

b) $\begin{cases} x = -\dfrac{\pi}{3} + 2k\pi \\ x = \dfrac{\pi}{9} + \dfrac{2k\pi}{3} \end{cases}$ $\quad k = 1 \quad (k = 0 \Rightarrow x < 0 !)$; $\quad k = 0, 1, 2$

$$S_p : \left\{ \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}, \frac{5\pi}{3} \right\}$$

c) $\begin{cases} x = -\dfrac{\pi}{60} + \dfrac{2k\pi}{5} \\ x = -\dfrac{7\pi}{60} + \dfrac{2k\pi}{5} \end{cases}$ $\quad k = 1, 2, 3, 4, 5$; $\quad k = 1, 2, 3, 4, 5$

$$S_p : \left\{ \frac{17\pi}{60}, \frac{23\pi}{60}, \frac{41\pi}{60}, \frac{47\pi}{60}, \frac{13\pi}{12}, \frac{71\pi}{60}, \frac{89\pi}{60}, \frac{19\pi}{12}, \frac{113\pi}{60}, \frac{119\pi}{60} \right\}$$

d) $\begin{cases} x = -\dfrac{\pi}{4} + 2k\pi \\ x = \dfrac{\pi}{4} + \dfrac{2k\pi}{5} \end{cases}$ $\quad k = 1$; $\quad k = 0, 1, 2, 3, 4$

$$S_p : \left\{ \frac{\pi}{4}, \frac{13\pi}{20}, \frac{21\pi}{20}, \frac{29\pi}{20}, \frac{7\pi}{4}, \frac{37\pi}{20} \right)$$

e) $x = \dfrac{5\pi}{12} + k\pi \qquad k = 0, 1$

$$S_p : \left\{ \frac{5\pi}{12}, \frac{17\pi}{12} \right\}$$

4 ~~3~~. Résoudre les équations suivantes (techniques diverses) + solution sur $]-\pi,\pi]$

(a) $\tan 3x = -1$

$\iff \tan 3x = \tan\left(-\frac{\pi}{4}\right)$

$\iff 3x = -\frac{\pi}{4} + k\pi \iff x = -\frac{\pi}{12} + \frac{k\pi}{3}$

$(k \in \mathbb{Z})$

$S_g : \left\{ -\frac{\pi}{12} + \frac{k\pi}{3} \;\middle|\; k \in \mathbb{Z} \right\}$

Sur $[0, 2\pi[$:

$S_p : \left\{ \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12} \right\}$

$k = 1, 2, 3, 4, 5, 6$

(b) $\cot 3x = \cot x$

CE $x \neq k\pi$

$\iff 3x = x + k\pi \iff 2x = k\pi \iff x = \frac{k\pi}{2}$

$\underset{CE}{\iff} x = \frac{(2k+1)\pi}{2}$

$S_g : \left\{ \frac{2k+1}{2}\pi \;\middle|\; k \in \mathbb{Z} \right\}$

Sur $[0, 2\pi[$

$S_p : \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$

$k = 0, 1$

(c) $\cos\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2}$

$\Leftrightarrow \cos\left(2x - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

$\Leftrightarrow \begin{cases} 2x - \frac{\pi}{3} = \frac{2\pi}{3} + 2k\pi \\ 2x - \frac{\pi}{3} = -\frac{2\pi}{3} + 2k\pi \end{cases}$

$\Leftrightarrow \begin{cases} 2x = \pi + 2k\pi \\ 2x = -\frac{\pi}{3} + 2k\pi \end{cases}$

$\Leftrightarrow \begin{cases} x = \frac{\pi}{2} + k\pi \\ x = -\frac{\pi}{6} + k\pi \end{cases}$

$S_g : \left\{ -\frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi \,\middle|\, k \in \mathbb{Z} \right\}$

$S_p : \left\{ \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\}$

$k = 0, 1$ et $k = 1, 2$

(d) $\sin x = \cos x$

$\Leftrightarrow \sin x = \sin\left(\frac{\pi}{2} - x\right)$

$\Leftrightarrow \begin{cases} x = \frac{\pi}{2} - x + 2k\pi \\ x = \pi - \left(\frac{\pi}{2} - x\right) + 2k\pi \end{cases}$

$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{2} + 2k\pi \\ 0 = \frac{\pi}{2} + 2k\pi \end{cases}$ (imp.)

$\Leftrightarrow x = \frac{\pi}{4} + k\pi$

$S_g : \left\{ \frac{\pi}{4} + k\pi \,\middle|\, k \in \mathbb{Z} \right\}$

Sur $[0, 2\pi[$

$S : \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$

$k = 0, 1$

(e) $\tan x \cot 4x = 1$

C.E. $x \neq \frac{\pi}{2} + k\pi$

$4x \neq k\pi \iff x \neq \frac{k\pi}{4}$

$\iff x \neq \frac{k\pi}{4}$

$\iff \tan x = \frac{1}{\cot 4x}$

$\iff \tan x = \tan 4x$

$\iff x = 4x + k\pi$

$\iff -3x = k\pi$

$\iff x = \frac{k\pi}{3}$

$S_g : \left\{ \frac{k\pi}{3} \mid k \in \mathbb{Z} \right\}$

$S_{in}\ ]-\pi, \pi]$

$S : \left\{ 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

$k = 0, 1, 2, 3, 4, 5$

(f) $3\tan^2 x = 1$

$\iff \tan x = \pm\frac{\sqrt{3}}{3}$

- $\tan x = \frac{\sqrt{3}}{3} \iff x = \frac{\pi}{6} + k\pi$ $\quad k = 0, 1$

- $\tan x = -\frac{\sqrt{3}}{3} \iff x = -\frac{\pi}{6} + k\pi$ $\quad k = 1, 2$

$S_g : \left\{ -\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi \mid k \in \mathbb{Z} \right\}$

$S_p : \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

(g) $2\cos^2 x = \cos x$

$\Leftrightarrow 2\cos^2 x - \cos x = 0 \Leftrightarrow \cos x\,(2\cos x - 1) = 0$

- $\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi$

- $\cos x = \frac{1}{2} \Leftrightarrow \begin{cases} x = \frac{\pi}{3} + 2k\pi \\ x = -\frac{\pi}{3} + 2k\pi \end{cases}$

$S_g : \left\{ -\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + k\pi \right\} k \in \mathbb{Z} \}$

$S_p : \left\{ \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3} \right\}$

(h) $2\sin^2 x - 3\sin x + 1 = 0$

On pose $y = \sin x \Rightarrow 2y^2 - 3y + 1 = 0$

$\Delta = 9 - 8 = 1 \qquad y_{1,2} = \frac{3 \pm 1}{4} \left\langle \begin{matrix} 1 \\ \frac{1}{2} \end{matrix} \right.$

- $\sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$

- $\sin x = \frac{1}{2} \Leftrightarrow \begin{cases} x = \frac{\pi}{6} + 2k\pi \\ x = \frac{5\pi}{6} + 2k\pi \end{cases}$

$S_g : \left\{ \frac{\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi, 5\frac{\pi}{6} + 2k\pi \right\}$

$S_p : \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$

(i) $3\cos^2 x = 7(1-\sin x)$

$\Leftrightarrow 3(1-\sin^2 x) = 7 - 7\sin x$

$\Leftrightarrow -3\sin^2 x + 7\sin x - 4 = 0$

On pose $y = \sin x \Rightarrow$ l'éq devient $-3y^2+7y-4=0$

$\Delta = 49 - 48 = 1 \Rightarrow y_{1,2} = \dfrac{-7 \pm 1}{-6} \left\langle \begin{array}{l} 1 \\ \frac{4}{3} \end{array} \right.$

- $\sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$
- $\sin x = \frac{4}{3} \Rightarrow$ imp

$S_g : \left\{ \frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z} \right\}$ et $S_p : \left\{ \frac{\pi}{2} \right\}$

(j) $\tan^2 x + \cot^2 x = 4$ $\quad$ CE : $\underbrace{x \neq \frac{\pi}{2} + k\pi,\ x \neq k\pi}_{x \neq \frac{k\pi}{2}}$

$\tan^2 x + \dfrac{1}{\tan^2 x} = 4$

$\Leftrightarrow \tan^4 x - 4\tan^2 x + 1 = 0$ (car $\tan x \neq 0$)

On pose $y = \tan^2 x \Rightarrow$ l'eq devient $y^2 - 4y + 1 = 0$

$\Delta = 12 \qquad y_{1,2} = \dfrac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

- $\tan^2 x = 2 - \sqrt{3} \Leftrightarrow \tan x \simeq \pm 0,518$ (*)
  $\Leftrightarrow x \simeq \tan^{-1}(0,518)$
  $\Leftrightarrow x \simeq \pm 0,478 + k\pi$
- $\tan^2 x = 2 + \sqrt{3} \Leftrightarrow \tan x \simeq \pm 1,932$
  $\Leftrightarrow x \simeq \pm 1,093 + k\pi$

(*) calculatrice

$$S_g : \left\{ \pm 0,478 + k\pi \; ; \; \pm 1,093 + k\pi \;\middle|\; k \in \mathbb{Z} \right\}$$

$$S_p : \{ 0,478 \; ; \; 1,093 \; ; \; 2,049 \; ; \; 2,664 \; ; \; 3,619 \; ; \; 4,235 \; ; \ldots$$

$$\ldots \; 5,190 \; ; \; 5,805 \}$$

(k) $\sin 2x + \tan 2x = 0$

$CE : 2x \neq \frac{\pi}{2} + k\pi \Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2}$

$\sin 2x + \frac{\sin 2x}{\cos 2x} = 0$

$\Leftrightarrow \sin 2x \left(1 + \frac{1}{\cos 2x}\right) = 0 \Leftrightarrow \sin 2x \frac{(1 + \cos 2x)}{\cancel{\cos 2x}} = 0$

$\neq 0$ à cause des CE

- $\sin 2x = 0 \Leftrightarrow 2x = k\pi$
  $\Leftrightarrow x = \frac{k\pi}{2}$
- $\cos 2x = -1 \Leftrightarrow 2x = \pi + 2k\pi$
  $\Leftrightarrow x = \frac{\pi}{2} + k\pi$

$S_g : \left\{ \frac{k\pi}{2} \mid k \in \mathbb{Z} \right\}$

$S_p : \left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$

(l) $\cos 2x = \cos x + 1$

$2\cos^2 x - 1 - \cos x - 1 = 0 \Leftrightarrow 2\cos^2 x - \cos x - 2 = 0$

$\Delta = 17 \qquad \cos x_{1,2} = \frac{1 \pm \sqrt{17}}{4} \begin{cases} \simeq 1{,}281 \\ \simeq -0{,}781 \end{cases}$

- $\cos x \simeq 1{,}281 \Rightarrow$ imp
- $\cos x \simeq -0{,}781 \Leftrightarrow \begin{cases} x \simeq 2{,}467 + 2k\pi \\ x \simeq -2{,}467 + 2k\pi \end{cases}$

$S_g : \left\{ -2{,}467 + 2k\pi ; 2{,}467 + 2k\pi \mid k \in \mathbb{Z} \right\}$

$S_p : \left\{ 2{,}467 ; 5{,}609 \right\}$

(m) $3\cos x(\sin x + \tan x) = 2(1+\cos x)$

CE : $x \neq \frac{\pi}{2} + k\pi$

$3\cos x\left(\sin x + \frac{\sin x}{\cos x}\right) = 2(1+\cos x)$

$\Leftrightarrow 3\sin x\cos x\left(1 + \frac{1}{\cos x}\right) = 2(1+\cos x)$

$\Leftrightarrow 3\sin x\cancel{\cos x}\,\frac{(1+\cos x)}{\cancel{\cos x}} = 2(1+\cos x)$

cf CE

$\Leftrightarrow 3\sin x(1+\cos x) = 2(1+\cos x)$

$\Leftrightarrow 3\sin x(1+\cos x) - 2(1+\cos x) = 0$

$\Leftrightarrow (1+\cos x)(3\sin x - 2) = 0$

(*) suite : voir page suivante

(n) $5\sin^2 x - 2\cos^2 x - 3\sin x\cos x = 0$

$\Leftrightarrow \frac{5\sin^2 x - 2\cos^2 x - 3\sin x\cos x}{\cos^2 x} = 0$ CE $x \neq \frac{\pi}{2} + k\pi$

$\Leftrightarrow 5\tan^2 x - 2 - 3\tan x = 0$

$\Delta = 9 + 40 = 49$ $\tan x_{1,2} = \frac{3 \pm 7}{10}$ $\begin{cases} 1 \\ -\frac{2}{5} \end{cases}$

. $\tan x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi$

. $\tan x = -\frac{2}{5} \Leftrightarrow x = -0{,}381 + k\pi$

$S_g : \{-0{,}381 + k\pi \,;\, \frac{\pi}{4} + k\pi \mid k \in \mathbb{Z}\}$

$S_p : \{\frac{\pi}{4} \,;\, 2{,}761 \,;\, \frac{3\pi}{4} \,;\, 5{,}902\}$

(*) • $1+\cos x = 0 \Leftrightarrow \cos x = -1 \Leftrightarrow x = \pi + 2k\pi$

• $3\sin x - 2 = 0 \Leftrightarrow \sin x = \frac{2}{3}$

$$\Leftrightarrow \begin{cases} x = 0{,}73 + 2k\pi \\ x = 2{,}41 + 2k\pi \end{cases}$$

$S_g : \{ 0{,}73 + 2k\pi ; 2{,}41 + 2k\pi ; \pi + 2k\pi \mid k \in \mathbb{Z} \}$

$S_p : \{ 0{,}73 ; 2{,}41 ; \pi \}$

(o) $\sin x + \cos x = 1$ On utilise les formules en $\tan \frac{x}{2}$ ($=t$)

$\Leftrightarrow \left(\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1 \right) (1+t^2)$

$\Leftrightarrow 2t + 1 - t^2 = 1 + t^2 \Leftrightarrow 2t - 2t^2 = 0 \Leftrightarrow 2t(1-t) = 0$

. $\tan \frac{x}{2} = 0 \Leftrightarrow \frac{x}{2} = k\pi \Leftrightarrow x = 2k\pi$

. $\tan \frac{x}{2} = 1 \Leftrightarrow \frac{x}{2} = \frac{\pi}{4} + k\pi \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$

$S_g : \left\{ \frac{\pi}{2} + 2k\pi , 2k\pi \right\} \; k \in \mathbb{Z}$

$S_p : \left\{ 0, \frac{\pi}{2} \right\}$

(p) $\cos x + \sqrt{3} \sin x = 1$ $t = \tan \frac{x}{2}$

$\Leftrightarrow \left(\frac{1-t^2}{1+t^2} + \sqrt{3} \frac{2t}{1+t^2} = 1 \right) . (1+t^2)$

$\Leftrightarrow 1 - t^2 + 2\sqrt{3}t = 1 + t^2 \Leftrightarrow 2\sqrt{3}t - 2t^2 = 0$

$\Leftrightarrow 2t(\sqrt{3} - t) = 0$

. $\tan \frac{x}{2} = 0 \Leftrightarrow \frac{x}{2} = k\pi \Leftrightarrow x = 2k\pi$

. $\tan \frac{x}{2} = \sqrt{3} \Leftrightarrow \frac{x}{2} = \frac{\pi}{3} + k\pi \Leftrightarrow x = \frac{2\pi}{3} + 2k\pi$

$S_g : \left\{ \frac{2\pi}{3} + 2k\pi , 2k\pi \right\} \; k \in \mathbb{Z}$

$S_p : \left\{ 0, \frac{2\pi}{3} \right\}$

(q) $\sqrt{2}\cos x - \sqrt{2}\sin x = 1$

$t = \tan\frac{x}{2}$

$\Leftrightarrow \sqrt{2}\,\frac{1-t^2}{1+t^2} - \sqrt{2}\,\frac{2t}{1+t^2} = 1$

$\Leftrightarrow \sqrt{2}\,(1-t^2) - 2\sqrt{2}\,t = 1 + t^2$

$\Leftrightarrow \sqrt{2} - \sqrt{2}\,t^2 - 2\sqrt{2}\,t - 1 - t^2 = 0 \Leftrightarrow (-\sqrt{2}-1)\,t^2 - 2\sqrt{2}\,t - 1 + \sqrt{2} = 0$

$\Delta = 8 - 4(-\sqrt{2}-1)(\sqrt{2}-1) = 8 + 4(\sqrt{2}+1)(\sqrt{2}-1)$

$= 8 + 4\,(1) = 12$

$\tan\frac{x_{1,2}}{2} = \frac{2\sqrt{2} \pm 2\sqrt{3}}{-2(\sqrt{2}+1)} = \frac{-\sqrt{2} \pm \sqrt{3}}{\sqrt{2}+1}$

$\tan\frac{x}{2}{}_1 = \frac{-\sqrt{2} + \sqrt{3}}{\sqrt{2}+1} \Leftrightarrow \frac{x}{2}{}_1 = \frac{\pi}{24} + k\pi$

$\Leftrightarrow x = \frac{\pi}{12} + 2k\pi$

(*) suite page suivante

(r) $2\sin x - 3\cos x = 3$

$t = \tan\frac{x}{2}$

$\Leftrightarrow \frac{4t}{1+t^2} - 3\,\frac{1-t^2}{1+t^2} = 3$

$\Leftrightarrow 4t - 3 + 3t^2 = 3 + 3t^2 \Leftrightarrow 4t - 6 = 0$

$\Leftrightarrow t = \frac{3}{2}$

$\tan\frac{x}{2} = \frac{3}{2} \Leftrightarrow \frac{x}{2} = 0{,}983 + k\pi \Leftrightarrow x = 1{,}966 + 2k\pi$

$S_g : \{ 1{,}966 + 2k\pi \mid k \in \mathbb{Z} \}$

$S_p : \{ 1{,}966 \}$

(7) $\tan \frac{x_2}{2} = \frac{-\sqrt{2}-\sqrt{3}}{\sqrt{2}+1} \Leftrightarrow \frac{x_2}{2} = -\frac{7\pi}{24} + k\pi$

$\Leftrightarrow x_2 = -\frac{7\pi}{12} + 2k\pi$

$S_g : \left\{ -\frac{7\pi}{12} + 2k\pi , \frac{\pi}{12} + 2k\pi \mid k \in \mathbb{Z} \right\}$

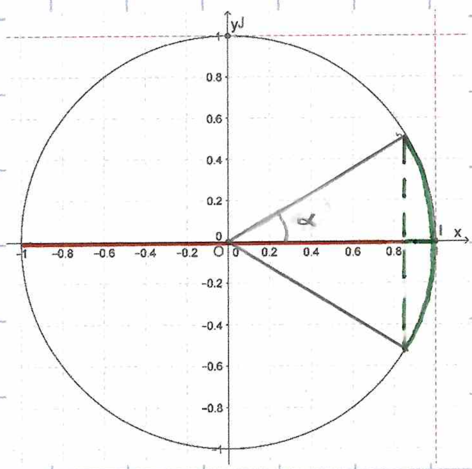
$S_p : \left\{ \frac{\pi}{12} , \frac{17\pi}{12} \right\}$

Inéquations trigonométriques : Solutions

0.1 Inéquations de base

Résoudre les inéquations suivantes et exprimer les solutions sur l'intervalle ~~$]-\pi, \pi]$~~ $[0, 2\pi[$:

1. $\cos x > \dfrac{\sqrt{3}}{2}$

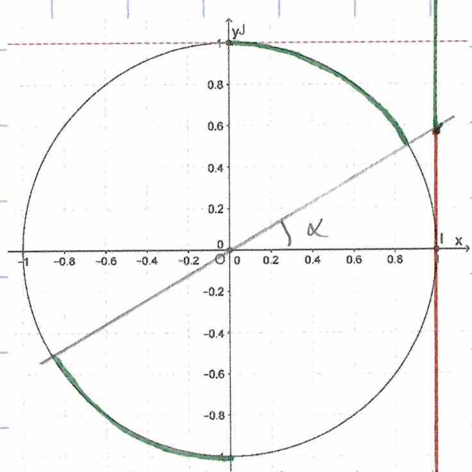


$\alpha = 30^\circ \ \left(\dfrac{\pi}{6}\right)$

$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ \left]0, \dfrac{\pi}{6}\right[\cup \left]\dfrac{11\pi}{6}, 2\pi\right[+ 2k\pi \right\}$

$S_p : \left]0, \dfrac{\pi}{6}\right[\cup \left]\dfrac{11 \cdot \pi}{6}, 2\pi\right[$

2. $3\tan y - \sqrt{3} \geq 0$ $\Rightarrow \tan y \geq \dfrac{\sqrt{3}}{3}$

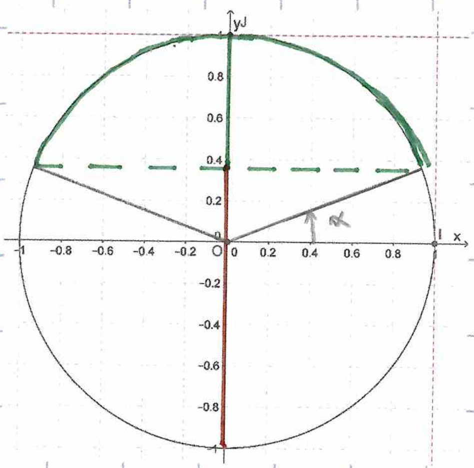


$\alpha = \dfrac{\pi}{6}$

$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ \left[\dfrac{\pi}{6}, \dfrac{\pi}{2}\right[+ k\pi \right\}$

$S_p : \left[\dfrac{\pi}{6}, \dfrac{\pi}{2}\right[\cup \left[\dfrac{7\pi}{6}, \dfrac{3\pi}{2}\right[$

3. $1 - 3\sin\left(x - \frac{\pi}{4}\right) \le 0$



$\Leftrightarrow \sin\left(x - \frac{\pi}{4}\right) \ge \frac{1}{3}$

$\alpha = \sin^{-1}\left(\frac{1}{3}\right) \approx 0,34$

$\Rightarrow 0,34 \le x - \frac{\pi}{4} \le 2,80$

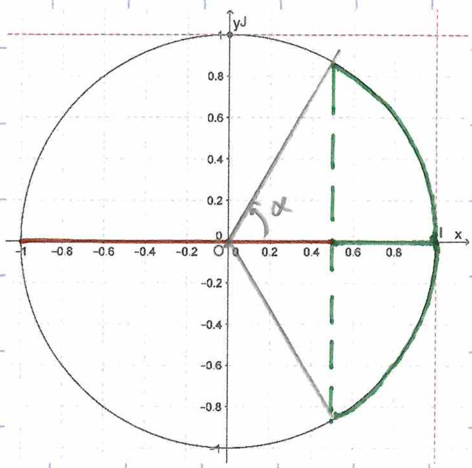
$\Leftrightarrow 1,125 \le x \le 3,586$

$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ [1,125 ; 3,586] + 2k\pi \right\}$

$S_p : [1,125 ; 3,586]$

4. $1 - 2\cos 5y < 0$

$\iff \cos 5y > \frac{1}{2}$



$\alpha = \frac{\pi}{3}$

$5y \in \left[0, \frac{\pi}{3}\right[\cup \left]\frac{5\pi}{3}, 2\pi\right[+ 2k\pi$

$\iff y \in \left[0, \frac{\pi}{15}\right[\cup \left]\frac{\pi}{3}, \frac{2\pi}{5}\right[+ \frac{2k\pi}{5}$

$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ \left[0, \frac{\pi}{15}\right[\cup \left]\frac{\pi}{3}, \frac{2\pi}{5}\right[+ \frac{2k\pi}{5} \right\}$

$S_p : \left[0, \frac{\pi}{15}\right[\cup \left]\frac{\pi}{3}, \frac{2\pi}{5}\right[\cup \left[\frac{2\pi}{5}, \frac{7\pi}{15}\right[\ldots$

$\ldots \cup \left]\frac{11\pi}{15}, \frac{4\pi}{5}\right[\cup \left[\frac{4\pi}{5}, \frac{13\pi}{15}\right[\cup \left]\frac{17\pi}{15}, \frac{6\pi}{5}\right[\ldots$

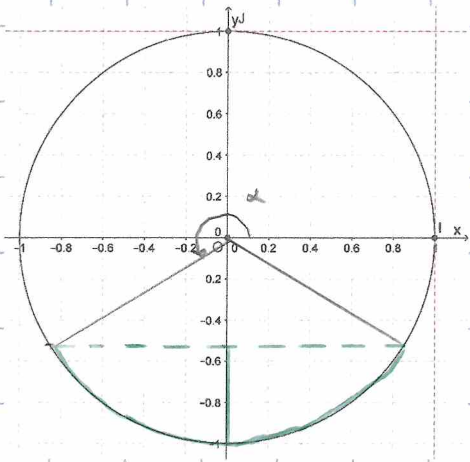
$\ldots \cup \left[\frac{6\pi}{5}, \frac{19\pi}{15}\right[\cup \left]\frac{23\pi}{5}, \frac{8\pi}{5}\right[\cup \left[\frac{8\pi}{5}, \frac{5\pi}{3}\right[\ldots$

$\ldots \cup \left]\frac{29\pi}{5}, 2\pi\right[$

ou $S_p : \left[0, \frac{\pi}{15}\right[\cup \left]\frac{\pi}{3}, \frac{7\pi}{15}\right[\cup \left]\frac{11\pi}{15}, \frac{13\pi}{5}\right[\ldots$

$\ldots \cup \left]\frac{17\pi}{15}, \frac{19\pi}{15}\right[\cup \left]\frac{23\pi}{5}, \frac{5\pi}{3}\right[\cup \left]\frac{29\pi}{5}, 2\pi\right[$

5. $2\sin\left(3t-\frac{\pi}{5}\right)+1<0$ $\Leftrightarrow \sin\left(3t-\frac{\pi}{5}\right) < -\frac{1}{2}$



$\alpha = \frac{7\pi}{6}$

$\Rightarrow \frac{7\pi}{6} < 3t - \frac{\pi}{5} < \frac{11\pi}{6}$

$\Leftrightarrow \frac{7\pi}{6} + \frac{\pi}{5} < 3t < \frac{11\pi}{6} + \frac{\pi}{5}$

$\Leftrightarrow \frac{41\pi}{30} < 3t < \frac{61\pi}{30} + 2k\pi$

$\Leftrightarrow \frac{41\pi}{90} < t < \frac{61\pi}{90} + \frac{2k\pi}{3}$

$S_g : \bigcup_{k\in\mathbb{Z}} \left\{ \left]\frac{41\pi}{90}, \frac{61\pi}{90}\right[+ \frac{2k\pi}{3} \right\}$ $\left(\frac{2\pi}{3} = \frac{60\pi}{90}\right)$

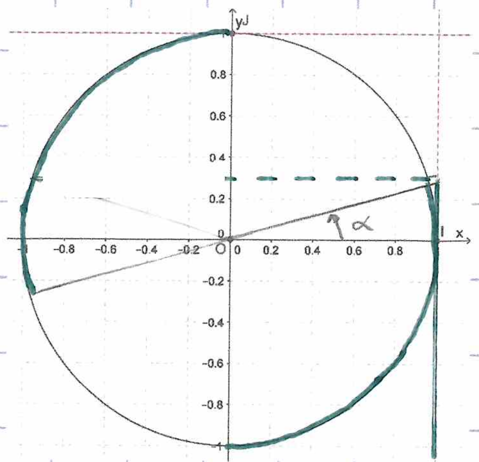
$S_p : \left]\frac{41\pi}{90}, \frac{61\pi}{90}\right[\cup \left]\frac{101\pi}{90}, \frac{121\pi}{90}\right[\ldots$

$\ldots \cup \left]\frac{161\pi}{90}, \underbrace{\frac{181\pi}{90}}_{>2\pi}\right[$ $\frac{181\pi}{90} - 2\pi = \frac{\pi}{90}$

$: \left[0, \frac{\pi}{90}\right[\cup \left]\frac{41\pi}{90}, \frac{61\pi}{90}\right[\ldots$

$\ldots \cup \left]\frac{101\pi}{90}, \frac{121\pi}{90}\right[\cup \left]\frac{161\pi}{90}, 2\pi\right[$

6. $3\tan 3x - 1 \leq 0$



$\Rightarrow \tan 3x \leq \frac{1}{3}$

$\alpha = \tan^{-1}\left(\frac{1}{3}\right) \approx 0{,}32$

$3x \in [0; 0{,}32] \cup]\frac{\pi}{2}, \pi] + k\pi$

$x \in [0; 0{,}107] \cup]\frac{\pi}{6}, \frac{\pi}{3}] + \frac{k\pi}{3}$

$S_g : \bigcup_{k\in\mathbb{Z}} \left\{ [0; 0{,}107] \cup]\frac{\pi}{6}, \frac{\pi}{3}] + \frac{k\pi}{3} \right\}$

S_p : $k=0$ $S_0 : [0; 0{,}107] \cup]\frac{\pi}{6}, \frac{\pi}{3}]$

$k=1$ $S_1 : [\frac{\pi}{3}; 1{,}154] \cup]\frac{\pi}{2}, \frac{2\pi}{3}]$

$k=2$ $S_2 : [\frac{2\pi}{3}; 2{,}202] \cup]\frac{5\pi}{6}, \pi]$

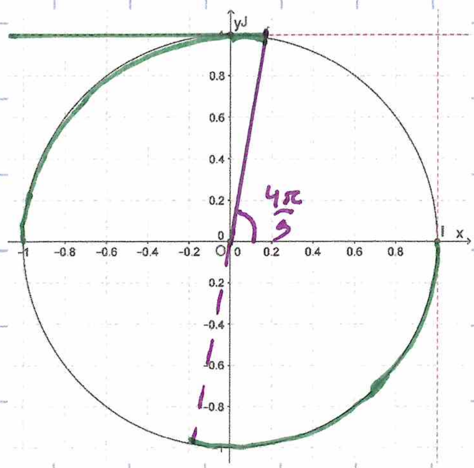
$k=3$ $S_3 : [\pi; 3{,}249] \cup]\frac{7\pi}{6}, \frac{4\pi}{3}]$

$k=4$ $S_4 : [\frac{4\pi}{3}; 4{,}296] \cup]\frac{3\pi}{2}, \frac{5\pi}{3}]$

$k=5$ $S_5 : [\frac{5\pi}{3}; 5{,}343] \cup]\frac{11\pi}{6}, 2\pi[$

$S_p = S_0 \cup S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$

7. $\cot\frac{4\pi}{9} - \cot\left(3x-\frac{\pi}{3}\right) \geq 0$ $\iff$ $\cot\left(3x-\frac{\pi}{3}\right) \leq \cot\frac{4\pi}{9}$



$$\frac{4\pi}{9} \leq 3x - \frac{\pi}{3} < \pi \qquad + k\pi$$

$$\Rightarrow \frac{4\pi}{9} + \frac{\pi}{3} \leq 3x < \pi + \frac{\pi}{3} \quad + k\pi$$

$$\iff \frac{7\pi}{9} \leq 3x < \frac{4\pi}{3} + k\pi$$

$$\iff \frac{7\pi}{27} \leq x < \frac{4\pi}{9} + \frac{k\pi}{3}$$

$$S_g: \bigcup_{k\in\mathbb{Z}} \left\{ \left[\frac{7\pi}{27} ; \frac{4\pi}{9}\right[+ \frac{k\pi}{3} \right\}$$

$\frac{9\pi}{27}$ / $\frac{3\pi}{9}$

S_p:

$k=0: \left[\frac{7\pi}{27}, \frac{4\pi}{9}\right[$	$k=1: \left[\frac{16\pi}{27}, \frac{7\pi}{9}\right[$
$k=2: \left[\frac{25\pi}{27}, \frac{10\pi}{9}\right[$	$k=3: \left[\frac{34\pi}{27}, \frac{13\pi}{9}\right[$
$k=4: \left[\frac{43\pi}{27}, \frac{16\pi}{9}\right[$	$k=5 \left[\frac{52\pi}{27}, \frac{19\pi}{9}\right[$

(*) $> 2\pi$

$$\Rightarrow \left[\frac{52\pi}{27}, 2\pi\right[\cup \left[0, \frac{\pi}{9}\right[$$

$$\Rightarrow S_p: \left[0, \frac{\pi}{9}\right[\cup \left[\frac{7\pi}{27}, \frac{4\pi}{9}\right[\cup \left[\frac{16\pi}{27}, \frac{7\pi}{9}\right[\cup \left[\frac{25\pi}{27}, \frac{10\pi}{9}\right[\ldots$$

$$\ldots \cup \left[\frac{34\pi}{27}, \frac{13\pi}{9}\right[\cup \left[\frac{43\pi}{27}, \frac{16\pi}{9}\right[\cup \left[\frac{52\pi}{27}, 2\pi\right[$$

0.2 Inéquations

Résoudre les inéquations suivantes et exprimer les solutions sur l'intervalle $]\cancel{-\pi}, \pi]$: $[0, 2\pi[$

1. $\sin^2 3x \le \dfrac{3}{4}$ $\iff \sin^2 3x - \dfrac{3}{4} \le 0$

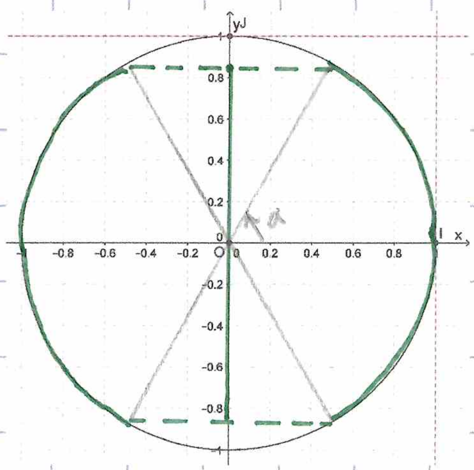
On pose $y = \sin 3x \Rightarrow y^2 - \dfrac{3}{4} \le 0$

y		$-\frac{\sqrt{3}}{2}$		$\frac{\sqrt{3}}{2}$	
In	+	0	−	0	+

$\Rightarrow -\dfrac{\sqrt{3}}{2} \le y \le \dfrac{\sqrt{3}}{2}$

$\iff -\dfrac{\sqrt{3}}{2} \le \sin 3x \le \dfrac{\sqrt{3}}{2}$

$\alpha = \dfrac{\pi}{3}$



$3x \in [0, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, \frac{4\pi}{3}] \ldots$

$\ldots \cup [\frac{5\pi}{3}, 2\pi[+ 2k\pi$

$\iff x \in [0, \frac{\pi}{9}] \cup [\frac{2\pi}{9}, \frac{4\pi}{9}] \cup [\frac{5\pi}{9}, \frac{2\pi}{3}[+ \dfrac{2k\pi}{3}$

$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ [0, \frac{\pi}{9}] \cup [\frac{2\pi}{9}, \frac{4\pi}{9}] \cup [\frac{5\pi}{9}, \frac{2\pi}{3}[+ \frac{2k\pi}{3} \right\}$

$\frac{6\pi}{9}$

S_p : $k=0$ $\quad [0, \frac{\pi}{9}] \cup [\frac{2\pi}{9}, \frac{4\pi}{9}] \cup [\frac{5\pi}{9}, \frac{2\pi}{3}[= S_1$

$k=1$ $\quad [\frac{2\pi}{3}, \frac{7\pi}{9}] \cup [\frac{8\pi}{9}, \frac{10\pi}{9}] \cup [\frac{11\pi}{9}, \frac{4\pi}{3}[= S_2$

$k=2$ $\quad [\frac{4\pi}{3}, \frac{13\pi}{9}] \cup [\frac{14\pi}{9}, \frac{16\pi}{9}] \cup [\frac{17\pi}{9}, 2\pi[= S_3$

$S_p = S_1 \cup S_2 \cup S_3$

2. $2\sin^2 x - 3\sin x + 1 < 0$

On pose $y = \sin x$

$2y^2 - 3y + 1 < 0$

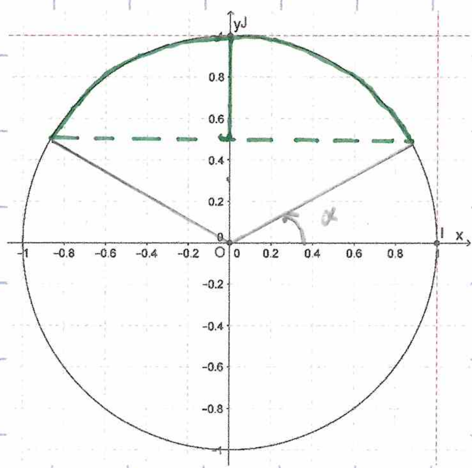
$\Delta = 9 - 8 = 1$

$y_{1,2} = \frac{3 \pm 1}{4} < \begin{matrix} 1 \\ \frac{1}{2} \end{matrix}$

y		$\frac{1}{2}$		1	
I_n	$+$	0	$-$	0	$+$

$\Rightarrow y \in \left]\frac{1}{2}, 1\right[$

$\Leftrightarrow \sin x \in \left]\frac{1}{2}, 1\right[$



$\alpha = \frac{\pi}{6}$

$x \in \left]\frac{\pi}{6}, \frac{\pi}{2}\right[\cup \left]\frac{\pi}{2}, \frac{5\pi}{6}\right[+ 2k\pi$

$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ \left]\frac{\pi}{6}, \frac{\pi}{2}\right[\cup \left]\frac{\pi}{2}, \frac{5\pi}{6}\right[+ 2k\pi \right\}$

$S_p : \left]\frac{\pi}{6}, \frac{\pi}{2}\right[\cup \left]\frac{\pi}{2}, \frac{5\pi}{6}\right[$

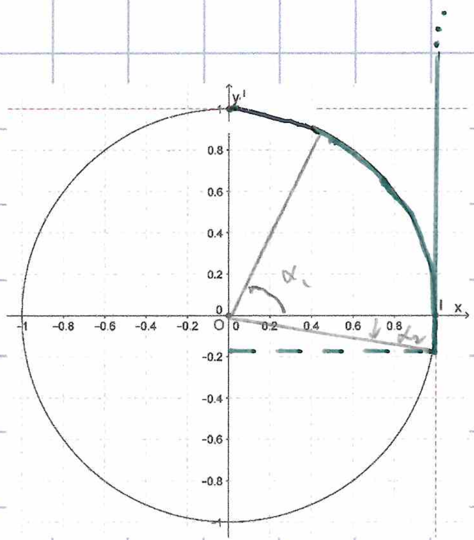
3. $3\sin x + 2\cos x > 1$ On pose $t = \tan\frac{x}{2}$. L'éq devient

$$\left(3\cdot\frac{2t}{1+t^2} + 2\,\frac{1-t^2}{1+t^2} > 1\right)(1+t^2) \quad (\text{car} > 0)$$

$$\Leftrightarrow 6t + 2 - 2t^2 > 1 + t^2 \Leftrightarrow -3t^2 + 6t + 1 > 0$$

$$\Delta = 36 + 12 = 48 \quad \text{et} \quad t_{1,2} = \frac{-6 \pm 4\sqrt{3}}{-6} = 1 \pm \frac{2\sqrt{3}}{3} \begin{cases} \simeq 2{,}15 \\ \simeq -0{,}15 \end{cases}$$

t		t_2		t_1	
I_n	$-$	0	$+$	0	$-$



$t \in \left]t_2, t_1\right[$

$\alpha_1 \simeq 1{,}136$ et $\alpha_2 \simeq -0{,}153$

$\Rightarrow \frac{x}{2} \in \left]-0{,}153 ; 1{,}136\right[+ k\pi$

$\Leftrightarrow x \in \left]-0{,}307 ; 2{,}273\right[+ 2k\pi$

$S_g : \bigcup_{k\in\mathbb{Z}} \left\{\left]-0{,}307 ; 2{,}273\right[+ 2k\pi\right\}$

$S_p : \left[0 ; 2{,}273\right[\cup \left]5{,}977 ; 2\pi\right[$

4. $\dfrac{\cos 2x}{1-2\sin 2x} \leq 0$

C'est une inég générale → T.S.

On cherche les zéros.

$\underline{N}$: $\cos 2x = 0 \iff 2x = \frac{\pi}{2} + k\pi$

$\iff x = \frac{\pi}{4} + \frac{k\pi}{2}$

→ sur $[0, 2\pi[$: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\underline{D}$: $1 - 2\sin 2x = 0 \iff \sin 2x = \frac{1}{2}$

$\iff \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 2x = \pi - \frac{\pi}{6} + 2k\pi \end{cases} \iff \begin{cases} x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{12} + k\pi \end{cases}$

→ sur $[0, 2\pi[$: $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

x	0		$\frac{\pi}{12}$		$\frac{\pi}{4}$		$\frac{5\pi}{12}$		$\frac{3\pi}{4}$		$\frac{13\pi}{12}$		$\frac{5\pi}{4}$		$\frac{17\pi}{12}$		$\frac{7\pi}{4}$		2π
$\cos 2x$		+		+	0	−	(*)	−	0	+		+	0	−		−	0	+	
$1-2\sin 2x$		+	0	−		−	0	+ (*)		+	0	−		−	0	+		+	
I_x		+	∄	−	0	+	∄	−	0	+	∄	−	0	+	∄	−	0	+	

$S_p : \left]\frac{\pi}{12}, \frac{\pi}{4}\right] \cup \left]\frac{5\pi}{12}, \frac{3\pi}{4}\right] \cup \left]\frac{13\pi}{12}, \frac{5\pi}{4}\right] \cup \left]\frac{17\pi}{12}, \frac{7\pi}{4}\right]$

$S_g = \bigcup_{k \in \mathbb{Z}} \left\{ S_p + 2k\pi \right\}$

(*) $\frac{\pi}{2} \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ et $\cos 2\frac{\pi}{2} = \cos \pi = -1 < 0$

(*) $\frac{\pi}{2} \in \left[\frac{5\pi}{12}, \frac{13\pi}{12}\right]$ et $1 - 2\sin 2\frac{\pi}{2} = 1 - 2\sin \pi = 1 > 0$

exercice diff ⇒ rigueur et lenteur

5. $1+\cos x > \sin^2 x$

$\iff 1+\cos x > 1-\cos^2 x \iff \cos^2 x+\cos x > 0$

$\iff \cos x\,(1+\cos x) > 0$

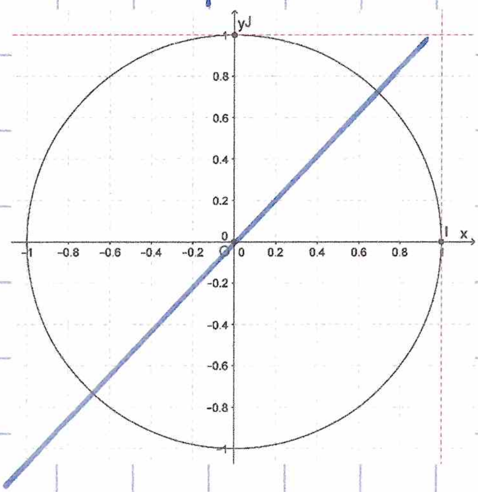
Zéros : • $\cos x = 0 \iff x = \frac{\pi}{2}+k\pi$

sur $[0, 2\pi[$: $\frac{\pi}{2}, \frac{3\pi}{2}$

• $1+\cos x = 0 \iff \cos x = -1 \iff x = \pi + 2k\pi$

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
$\cos x$		$+$	0	$-$		$-$	0	$+$	
$1+\cos x$		$+$		$+$	0	$+$		$+$	
		$+$	0	$-$	0	$-$	0	$+$	

(*)



$S_p : [0, \frac{\pi}{2}[\cup]\frac{3\pi}{2}, 2\pi[$

$S_{\mathbb{R}} = \bigcup_{k\in\mathbb{Z}} \{ S_p + 2k\pi \}$

(*) Rappel : c'est une des exceptions des T.S.

$-1 \leq \cos x \leq 1 \iff -1+1 \leq \cos x + 1 \leq 1+1$

$\iff 0 \leq \cos x + 1 \leq 2 \iff \cos x + 1 \geq 0$!

(Idem pour $\cos x - 1$, $\sin x - 1$, $\sin x + 1$, $1-\cos x$,
$1+\cos x$, $1-\sin x$ et $1+\sin x$)

6. $\dfrac{2\sin 2x - 1}{\cos 2x - 3\cos x + 2} > 0$

Pour faire le tableau de signe, on cherche les zéros

$\underline{N}$: $2\sin 2x - 1 = 0 \iff \sin 2x = \dfrac{1}{2}$

$\iff \begin{cases} 2x = \dfrac{\pi}{6} + 2k\pi \\ 2x = \dfrac{5\pi}{6} + 2k\pi \end{cases} \iff \begin{cases} x = \dfrac{\pi}{12} + k\pi \\ x = \dfrac{5\pi}{12} + k\pi \end{cases}$

Sur $[0; 2\pi[$, les solutions sont : $\left\{ \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{13\pi}{12}, \dfrac{17\pi}{12} \right\}$

$\underline{D}$: $\cos 2x - 3\cos x + 2 = 0 \iff 2\cos^2 x - 1 - 3\cos x + 2 = 0$

$\iff 2\cos^2 x - 3\cos x + 1 = 0$

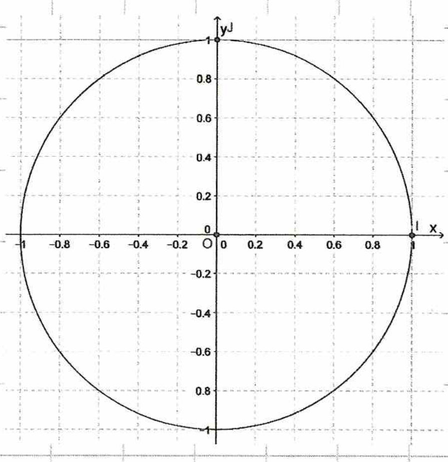
$\Delta = 1$, $\cos x_{1,2} = \dfrac{3 \pm 1}{4} \left\langle \begin{array}{l} 1 \\ \frac{1}{2} \end{array} \right.$

$\cos x = 1 \iff x = 2k\pi$

$\cos x = \dfrac{1}{2} \iff x = \pm \dfrac{\pi}{3} + 2k\pi$

$\underline{\text{Rmq}}$: D se factorise en

$2\left(\cos x - \dfrac{1}{2}\right)(\cos x - 1)$



Sur $[0, 2\pi[$, les solutions sont $\left\{ 0, \dfrac{\pi}{3}, \dfrac{5\pi}{3} \right\}$

x	0		$\frac{\pi}{12}$		$\frac{\pi}{3}$		$\frac{5\pi}{12}$		$\frac{13\pi}{12}$		$\frac{17\pi}{12}$		$\frac{5\pi}{3}$		2π
N		$-$	0	$+$		$+$	0	$-$	0	$+$	0	$-$		$-$	
$2\cos x - 1$		$+$		$+$	0	$-$		$-$		$-$		$-$	0	$+$	
$\cos x - 1$	0	$-$		$-$		$-$		$-$		$-$		$-$		$-$	
I_n	$\nexists$	$+$	0	$-$	$\nexists$	$+$	0	$-$	0	$+$	0	$-$	$\nexists$	$+$	

$$S_p : \left]0, \frac{\pi}{12}\right[\cup \left]\frac{\pi}{3}, \frac{5\pi}{12}\right[\cup \left]\frac{13\pi}{12}, \frac{17\pi}{12}\right[\cup \left]\frac{5\pi}{3}, 2\pi\right[$$

$$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ S_p + 2k\pi \right\}$$

7. $\tan^2 2x + (1-\sqrt{3})\tan 2x - \sqrt{3} \le 0$

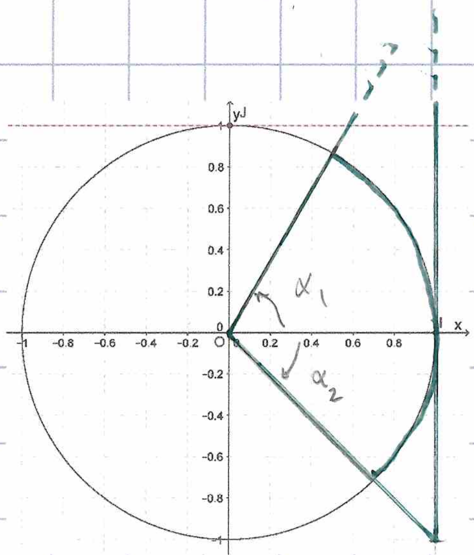
On pose $y = \tan 2x$

$$y^2 + (1-\sqrt{3})y - \sqrt{3} \le 0$$

$$\begin{aligned}\Delta &= (1-\sqrt{3})^2 + 4\sqrt{3} \\ &= 1 + 3 - 2\sqrt{3} + 4\sqrt{3} \\ &= 1 + 3 + 2\sqrt{3} \\ &= (1+\sqrt{3})^2\end{aligned}$$

$$y_{1,2} = \frac{-1+\sqrt{3} \pm (1+\sqrt{3})}{2} \begin{cases} \sqrt{3} \\ -1 \end{cases}$$

y		-1		$\sqrt{3}$	
Im	$+$	0	$-$	0	$+$



$y \in [-1, \sqrt{3}]$

$\tan 2x \in [-1, \sqrt{3}]$

$\alpha_1 = \frac{\pi}{3}$ et $\alpha_2 = -\frac{\pi}{4} = \frac{7\pi}{4}$

$\cancel{2x \in [0, \frac{\pi}{3}] \cup [\frac{7\pi}{4}, 2\pi[+ k\pi}$

$\cancel{\Leftrightarrow x \in [0, \frac{\pi}{6}] \cup [\frac{7\pi}{8}, \pi[+ \frac{k\pi}{2}}$

$\cancel{S_g : \bigcup_{k\in\mathbb{Z}} \left\{ [0, \frac{\pi}{6}] \cup [\frac{7\pi}{8}, \pi[+ \frac{k\pi}{2} \right\}}$

$2x \in [-\frac{\pi}{4}, \frac{\pi}{3}] + k\pi \Leftrightarrow x \in [-\frac{\pi}{8}, \frac{\pi}{6}] + \frac{k\pi}{2}$

$$S_g : \bigcup_{k\in\mathbb{Z}} \left\{ [-\frac{\pi}{8}, \frac{\pi}{6}] + \frac{k\pi}{2} \right\}$$

(*)

S_p : $k=0$ $\quad S_0 : [0, \frac{\pi}{6}] \cup [\frac{5\pi}{8}, \pi[$

$k=1$ $\quad S_1 : [\frac{\pi}{2}, \frac{2\pi}{3}] \cup [\frac{9\pi}{8}, \frac{3\pi}{2}[$

$k=2$ $\quad S_2 : [\pi, \frac{7\pi}{6}] \cup [\frac{13\pi}{8}, 2\pi[$

$k=3$ $\quad S_3 : [\frac{3\pi}{2}, \frac{5\pi}{3}] \cup [\underbrace{\frac{17\pi}{8}, \frac{5\pi}{2}}_{> 2\pi}[$

$\Rightarrow [\frac{\pi}{8}, \frac{\pi}{2}[$

(on peut modifier (*)

$S_p : S_0 \cup S_1 \cup S_2 \cup S_3$

S_p : $k=0$ $\quad S_0 : [-\frac{\pi}{8}, \frac{\pi}{6}]$, $[0, \frac{\pi}{6}] \cup [\frac{15\pi}{8}, 2\pi[$

$k=1$ $\quad S_1 : [\frac{3\pi}{8}, \frac{2\pi}{3}]$

$k=2$ $\quad S_2 : [\frac{7\pi}{8}, \frac{7\pi}{6}]$

$k=3$ $\quad S_3 : [\frac{11\pi}{8}, \frac{5\pi}{3}]$

$S = S_0 \cup S_1 \cup S_2 \cup S_3$

8. $\sin 3x - \sin 2x + \sin x \geq 0$ ⊛ Simpson

$\Leftrightarrow 2 \sin 2x \cos x - \sin 2x \geq 0$

$\Leftrightarrow \sin 2x \, (2\cos x - 1) \geq 0$

zéros : • $\sin 2x = 0 \Leftrightarrow 2x = k\pi \Leftrightarrow x = \dfrac{k\pi}{2}$

sur $[0, 2\pi[$: $0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$

• $2\cos x - 1 = 0 \Leftrightarrow \cos x = \dfrac{1}{2} \Leftrightarrow x = \pm\dfrac{\pi}{3} + 2k\pi$

sur $[0, 2\pi]$: $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$

x	0		$\frac{\pi}{3}$		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		$\frac{5\pi}{3}$		2π
$\sin 2x$	0	+		+	0	−	0	+	0	−		−	0
$2\cos x - 1$		+	0	−		−		−		−	0	+	
I_n	0	+	0	−	0	+	0	−	0	+	0	−	0

$S_p : \left[0, \dfrac{\pi}{3}\right] \cup \left[\dfrac{\pi}{2}, \pi\right] \cup \left[\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right]$

$S_g : \bigcup\limits_{k \in \mathbb{Z}} \left\{ S_p + 2k\pi \right\}$