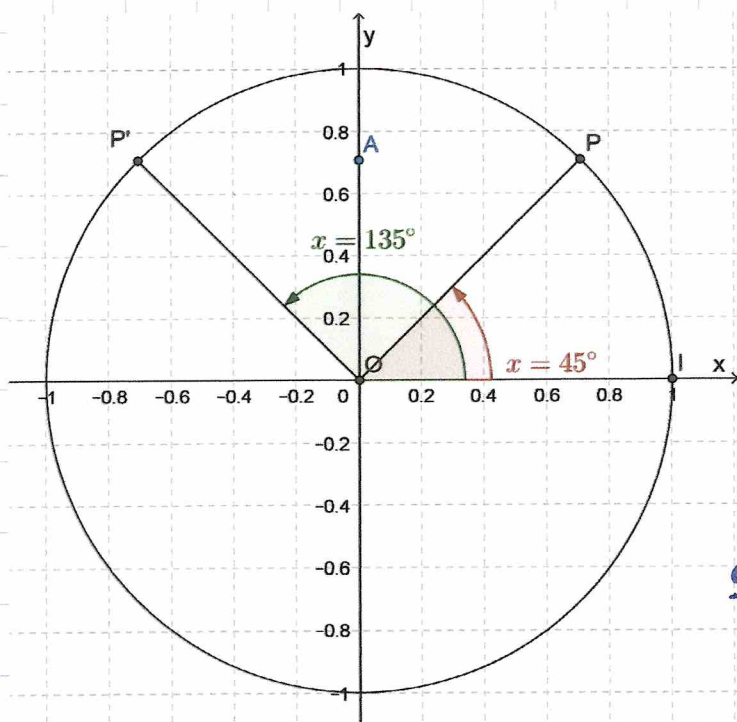


Equations trigonométriques : Solutions

1. Résoudre graphiquement les équations suivantes :

(a) $\sin x = \frac{\sqrt{2}}{2}$



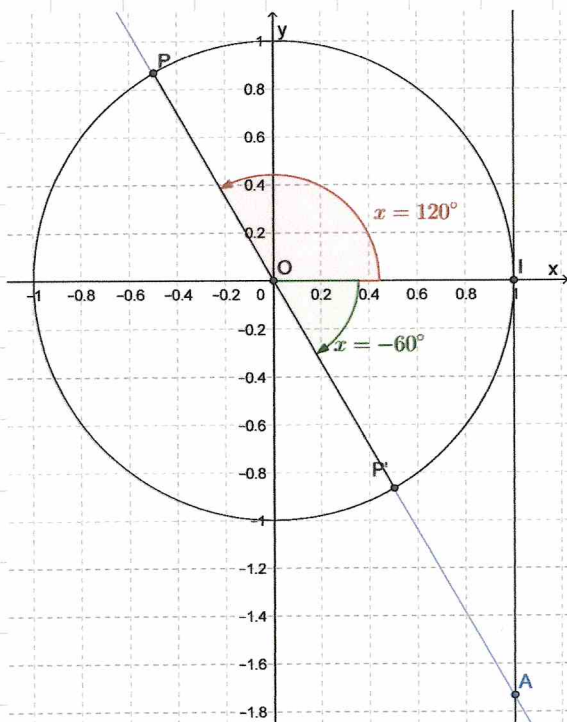
$\begin{cases} x = 45^\circ \\ x = 135^\circ \end{cases}$ lect. graph

$\Leftrightarrow \begin{cases} x = \frac{\pi}{4} + 2k\pi \\ x = \frac{3\pi}{4} + 2k\pi \end{cases}$

$\odot 180^\circ = \pi \text{ rad}$
(règle de 3)

$S_g : \left\{ \frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi \right\}$
 $\{ k \in \mathbb{Z} \}$

(b) $\tan x = -\sqrt{3}$



$\begin{cases} x = 120^\circ \\ x = -60^\circ \end{cases}$

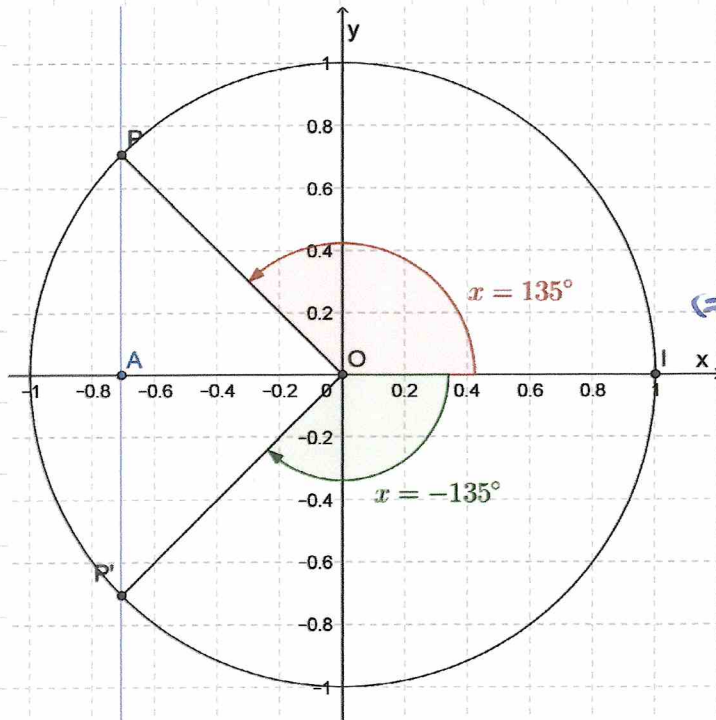
$\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ x = -\frac{\pi}{3} + 2k\pi \end{cases}$

ou $\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ x = \frac{5\pi}{3} + 2k\pi \end{cases}$

ou $x = \frac{2\pi}{3} + k\pi$

$S_g : \left\{ \frac{2\pi}{3} + k\pi \right\}$ $\{ k \in \mathbb{Z} \}$

(c) $1 + \sqrt{2} \cos x = 0$

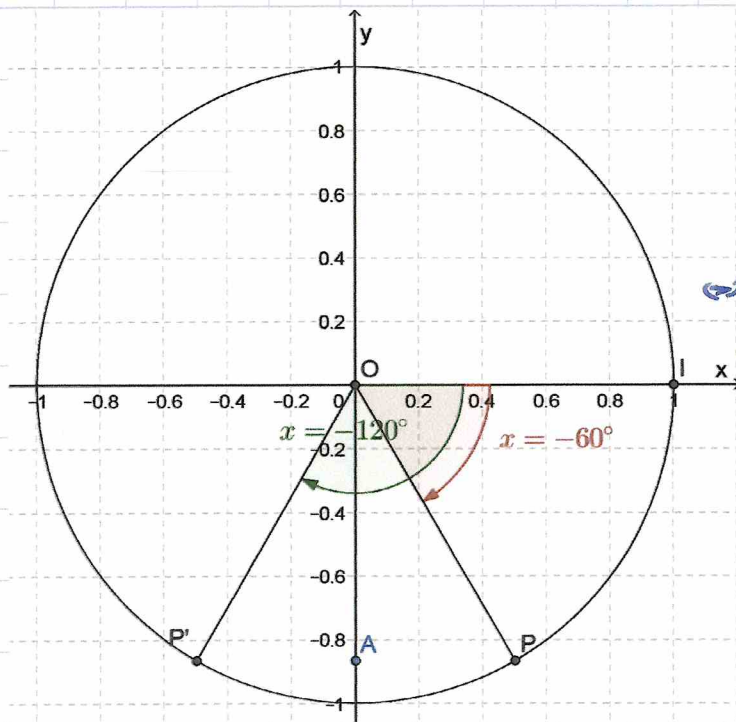


$$\begin{cases} x = 135^\circ \\ x = -135^\circ \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{3\pi}{4} + 2k\pi \\ x = -\frac{3\pi}{4} + 2k\pi \end{cases}$$

$$S_g: \left\{ \pm \frac{3\pi}{4} + 2k\pi \mid k \in \mathbb{Z} \right\}$$

(d) $2\sqrt{3} \sin x + 3 = 0$



$$\begin{cases} x = -60^\circ \\ x = -120^\circ \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ x = -\frac{2\pi}{3} + 2k\pi \end{cases}$$

$$\text{ou}$$

$$\begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ x = \frac{4\pi}{3} + 2k\pi \end{cases}$$

$$S_g: \left\{ -\frac{\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\}$$

2. Résoudre les équations suivantes

$$(a) \sin x = \frac{1}{4}$$

$$\arcsin \frac{1}{4} \approx 0,253$$

$$\begin{cases} x \approx 0,253 + 2k\pi \\ x \approx 2,889 + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$$

$$S_g = \{ 0,253 + 2k\pi; 2,889 + 2k\pi \mid k \in \mathbb{Z} \}$$

$$(b) \frac{\cot x}{5} - \frac{\cot x}{3} = \frac{1}{4}$$

$$\Leftrightarrow \frac{-2 \cot x}{15} = \frac{1}{4} \Leftrightarrow \cot x = -\frac{15}{8} \Leftrightarrow \tan x = -\frac{8}{15}$$

$$\arctan \left(-\frac{8}{15} \right) \approx -0,49$$

$$x \approx -0,49 + k\pi \quad (k \in \mathbb{Z})$$

$$S_g = \{ -0,49 + k\pi \mid k \in \mathbb{Z} \}$$

$$(c) \sqrt{12} \cos x = \cos x - 2$$

$$\Leftrightarrow (\sqrt{12} - 1) \cos x = -2 \Leftrightarrow \cos x = \frac{-2}{\sqrt{12} - 1}$$

$$\arccos \left(\frac{-2}{\sqrt{12} - 1} \right) \approx 2,518$$

$$\begin{cases} x \approx 2,518 + 2k\pi \\ x \approx -2,518 + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$$

$$S_g = \{ \pm 2,518 + 2k\pi \mid k \in \mathbb{Z} \}$$

3. Résoudre les équations suivantes :

(a) $\sin 3x = \sin x$

$$\begin{cases} 3x = x + 2k\pi \\ 3x = \pi - x + 2k\pi \end{cases} \Leftrightarrow \begin{cases} 2x = 2k\pi \\ 4x = \pi + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = k\pi \\ x = \frac{\pi}{4} + k\frac{\pi}{2} \end{cases} \quad S_g : \left\{ k\pi, \frac{\pi}{4} + k\frac{\pi}{2} \mid k \in \mathbb{Z} \right\}$$

(b) $\cos 2x = \cos \left(x - \frac{\pi}{3} \right)$

$$\begin{cases} 2x = x - \frac{\pi}{3} + 2k\pi \\ 2x = -\left(x - \frac{\pi}{3}\right) + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ 3x = \frac{\pi}{3} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ x = \frac{\pi}{9} + \frac{2k\pi}{3} \end{cases} \quad S_g : \left\{ -\frac{\pi}{3} + 2k\pi, \frac{\pi}{9} + \frac{4\pi}{3} \mid k \in \mathbb{Z} \right\}$$

(c) $\cos \left(5x + \frac{\pi}{3} \right) = \frac{\sqrt{2}}{2}$

$$\begin{cases} 5x + \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi \\ 5x + \frac{\pi}{3} = -\frac{\pi}{4} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 5x = -\frac{\pi}{12} + 2k\pi \\ 5x = -\frac{7\pi}{12} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{60} + \frac{2k\pi}{5} \\ x = -\frac{7\pi}{60} + \frac{2k\pi}{5} \end{cases} \quad S_g : \left\{ -\frac{\pi}{60} + 2k\frac{\pi}{5}, -\frac{7\pi}{60} + \frac{2k\pi}{5} \mid k \in \mathbb{Z} \right\}$$

$$(d) \sin 3x = \sin \left(2x - \frac{\pi}{4} \right)$$

$$\begin{cases} 3x = 2x - \frac{\pi}{4} + 2k\pi \\ 3x = \pi - \left(2x - \frac{\pi}{4} \right) + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + 2k\pi \\ 5x = \frac{5\pi}{4} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{4} + 2k\pi \\ x = \frac{\pi}{4} + \frac{2k\pi}{5} \end{cases}$$

$$S_g : \left\{ -\frac{\pi}{4} + 2k\pi, \frac{\pi}{4} + \frac{2k\pi}{5} \mid k \in \mathbb{Z} \right\}$$

$$(e) \tan \left(-x + \frac{\pi}{4} \right) = \frac{-\sqrt{3}}{3}$$

$$-x + \frac{\pi}{4} = -\frac{\pi}{6} + k\pi$$

$$\Leftrightarrow -x = -\frac{5\pi}{12} + k\pi \Leftrightarrow x = \frac{5\pi}{12} + k\pi$$

$$S_g : \left\{ \frac{5\pi}{12} + k\pi \mid k \in \mathbb{Z} \right\}$$

Après avoir résolu ces équations, exprimer explicitement les solutions comprises dans l'intervalle $]-\pi, \pi[$ $[0, 2\pi[$

$$\begin{array}{lll} a) \begin{cases} x = k\pi & (*)_1 & k=0,1 & x=0, \pi \\ x = \frac{\pi}{4} + k\frac{\pi}{2} & (*)_2 & k=0,1,2,3 & x=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \end{cases} \end{array}$$

$(*)_1$ Pour trouver les des sol, il faut donner 2 valeurs à k (2 π)

$(*)_2$ Pour trouver les les solutions, il faut donner 4 valeurs à k (4 $\frac{\pi}{2} = 2\pi$)

$$S_p : \left\{ 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

$$b) \begin{cases} x = -\frac{\pi}{3} + 2k\pi \\ x = \frac{\pi}{9} + \frac{2k\pi}{3} \end{cases}$$

$$k=1 \quad (k=0 \Rightarrow x < 0!)$$

$$k=0, 1, 2$$

$$S_P: \left\{ \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}, \frac{5\pi}{3} \right\}$$

$$c) \begin{cases} x = -\frac{\pi}{60} + \frac{2k\pi}{5} \\ x = -\frac{7\pi}{60} + \frac{2k\pi}{5} \end{cases}$$

$$k=1, 2, 3, 4, 5$$

$$k=1, 2, 3, 4, 5$$

$$S_P: \left\{ \frac{17\pi}{60}, \frac{23\pi}{60}, \frac{41\pi}{60}, \frac{47\pi}{60}, \frac{13\pi}{12}, \frac{7\pi}{60}, \frac{89\pi}{60}, \frac{19\pi}{12}, \frac{113\pi}{60}, \frac{119\pi}{60} \right\}$$

$$d) \begin{cases} x = -\frac{\pi}{4} + 2k\pi \\ x = \frac{\pi}{4} + \frac{2k\pi}{5} \end{cases}$$

$$k=1$$

$$k=0, 1, 2, 3, 4$$

$$S_P: \left\{ \frac{\pi}{4}, \frac{13\pi}{20}, \frac{21\pi}{20}, \frac{29\pi}{20}, \frac{7\pi}{4}, \frac{37\pi}{20} \right\}$$

$$e) x = \frac{5\pi}{12} + k\pi$$

$$k=0, 1$$

$$S_P: \left\{ \frac{5\pi}{12}, \frac{17\pi}{12} \right\}$$

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3. Résoudre les équations suivantes (techniques diverses) + solutions sur $]-\pi, \pi]$

(a) $\tan 3x = -1$

$$\Leftrightarrow \tan 3x = \tan\left(-\frac{\pi}{4}\right)$$

$$\Leftrightarrow 3x = -\frac{\pi}{4} + k\pi \Leftrightarrow x = -\frac{\pi}{12} + \frac{k\pi}{3} \quad (k \in \mathbb{Z})$$

$$S_g: \left\{ -\frac{\pi}{12} + \frac{k\pi}{3} \mid k \in \mathbb{Z} \right\}$$

$$S_{\text{sur } [0, 2\pi[}:$$

$$S_p: \left\{ \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12} \right\}$$

$$k = 1, 2, 3, 4, 5, 6$$

(b) $\cot 3x = \cot x$

$$c \in x \neq k\pi$$

$$\Leftrightarrow 3x = x + k\pi \Leftrightarrow 2x = k\pi \Leftrightarrow x = \frac{k\pi}{2}$$

$$\Leftrightarrow x = \frac{(2k+1)\pi}{2}$$

$$S_g: \left\{ \frac{2k+1}{2} \pi \mid k \in \mathbb{Z} \right\}$$

$$S_{\text{sur } [0, 2\pi[}$$

$$S_p: \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$k = 0, 1$$

$$(c) \cos\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2}$$

$$\Leftrightarrow \cos\left(2x - \frac{\pi}{3}\right) = \cos\left(-\frac{2\pi}{3}\right)$$

$$\Leftrightarrow \begin{cases} 2x - \frac{\pi}{3} = \frac{-2\pi}{3} + 2k\pi \\ 2x - \frac{\pi}{3} = \frac{2\pi}{3} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x = \pi + 2k\pi \\ 2x = \frac{\pi}{3} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{2} + k\pi \\ x = -\frac{\pi}{6} + k\pi \end{cases} \quad \begin{aligned} S_g &: \left\{ -\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\} \\ S_p &: \left\{ \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\} \end{aligned}$$

$$(d) \sin x = \cos x$$

$$k=0,1 \quad \text{or} \quad k=1,2$$

$$\Leftrightarrow \sin x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{2} - x + 2k\pi \\ x = \pi - \left(\frac{\pi}{2} - x\right) + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{2} + 2k\pi \\ 0 = \frac{\pi}{2} + 2k\pi \quad (\text{imp.}) \end{cases}$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi \quad S_g: \left\{ \frac{\pi}{4} + k\pi \mid k \in \mathbb{Z} \right\}$$

$$S_m \cap [0, 2\pi[$$

$$S: \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\} \quad k=0, 1$$

$$(e) \tan x \cot 4x = 1$$

$$CE: x \neq \frac{\pi}{2} + k\pi$$

$$\Leftrightarrow \tan x = \frac{1}{\cot 4x}$$

$$4x \neq k\pi \Leftrightarrow x \neq \frac{k\pi}{4}$$

$$\Leftrightarrow x \neq \frac{k\pi}{4}$$

$$\Leftrightarrow \tan x = \tan 4x$$

$$\Leftrightarrow x = 4x + k\pi$$

$$\Leftrightarrow -3x = k\pi$$

$$\Leftrightarrow x = \frac{k\pi}{3}$$

$$S_{\text{un}}]-\pi, \pi]$$

$$S: \left\{ 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

$$S_g: \left\{ \frac{k\pi}{3} \mid k \in \mathbb{Z} \right\}$$

$$k = 0, 1, 2, 3, 4, 5$$

$$(f) 3 \tan^2 x = 1$$

$$\Leftrightarrow \tan x = \pm \frac{\sqrt{3}}{3}$$

$$\bullet \tan x = \frac{\sqrt{3}}{3} \Leftrightarrow x = \frac{\pi}{6} + k\pi \quad k = 0, 1$$

$$\bullet \tan x = -\frac{\sqrt{3}}{3} \Leftrightarrow x = -\frac{\pi}{6} + k\pi \quad k = 1, 2$$

$$S_g: \left\{ -\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi \mid k \in \mathbb{Z} \right\}$$

$$S_p: \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$(g) 2 \cos^2 x = \cos x$$

$$\Leftrightarrow 2 \cos^2 x - \cos x = 0 \quad (\Leftrightarrow) \cos x (2 \cos x - 1) = 0$$

$$\bullet \cos x = 0 \quad (\Leftrightarrow) \quad x = \frac{\pi}{2} + k\pi$$

$$\bullet \cos x = \frac{1}{2} \quad (\Leftrightarrow) \quad \begin{cases} x = \frac{\pi}{3} + 2k\pi \\ x = -\frac{\pi}{3} + 2k\pi \end{cases}$$

$$S_g : \left\{ -\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + k\pi \right\} \quad k \in \mathbb{Z}$$

$$S_r : \left\{ \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{3}, \frac{5\pi}{6} \right\}$$

$$(h) 2 \sin^2 x - 3 \sin x + 1 = 0$$

$$\text{On pose } y = \sin x \Rightarrow 2y^2 - 3y + 1 = 0$$

$$\Delta = 9 - 8 = 1 \quad y_{1,2} = \frac{3 \pm 1}{4} < \begin{matrix} 1 \\ \frac{1}{2} \end{matrix}$$

$$\bullet \sin x = 1 \quad (\Leftrightarrow) \quad x = \frac{\pi}{2} + 2k\pi$$

$$\bullet \sin x = \frac{1}{2} \quad (\Leftrightarrow) \quad \begin{cases} x = \frac{\pi}{6} + 2k\pi \\ x = \frac{5\pi}{6} + 2k\pi \end{cases}$$

$$S_g : \left\{ \frac{\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi, \frac{5\pi}{6} + 2k\pi \right\}$$

$$S_r : \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$$

$$(i) 3 \cos^2 x = 7(1 - \sin x)$$

$$\Leftrightarrow 3(1 - \sin^2 x) = 7 - 7 \sin x$$

$$\Leftrightarrow -3 \sin^2 x + 7 \sin x - 4 = 0$$

On pose $y = \sin x \rightarrow$ l'éq devient $3y^2 + 7y - 4 = 0$

$$\Delta = 49 - 48 = 1 \Rightarrow y_{1,2} = \frac{-7 \pm 1}{-6} \begin{matrix} \swarrow \\ \searrow \end{matrix} \begin{matrix} 1 \\ \frac{4}{3} \end{matrix}$$

$$\bullet \sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$$

$$\bullet \sin x = \frac{4}{3} \rightarrow \text{imp}$$

$$S_g : \left\{ \frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z} \right\} \quad \text{et} \quad S_p : \left\{ \frac{\pi}{2} \right\}$$

$$(j) \tan^2 x + \cot^2 x = 4$$

$$\underline{CE} : \underbrace{x \neq \frac{\pi}{2} + k\pi, x \neq k\pi}_{x \neq \frac{k\pi}{2}}$$

$$\tan^2 x + \frac{1}{\tan^2 x} = 4$$

$$\Leftrightarrow \tan^4 x - 4 \tan^2 x + 1 = 0 \quad (\text{car } \tan x \neq 0)$$

On pose $y = \tan^2 x \rightarrow$ l'éq devient $y^2 - 4y + 1 = 0$

$$\Delta = 12 \quad y_{1,2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$$\bullet \tan^2 x = 2 - \sqrt{3} \Leftrightarrow \tan x \approx \pm 0,518$$

$$\Leftrightarrow x \approx \tan^{-1}(0,518) \quad \text{ⓧ}$$

$$\Leftrightarrow x \approx \pm 0,478 + k\pi$$

$$\bullet \tan^2 x = 2 + \sqrt{3} \Leftrightarrow \tan x \approx \pm 1,932$$

$$\Leftrightarrow x \approx \pm 1,093 + k\pi$$

ⓧ calculatrice

$$S_g : \{ \pm 0,478 + k\pi ; \pm 1,093 + k\pi \} \{ k \in \mathbb{Z} \}$$

$$S_p : \{ 0,478 ; 1,093 ; 2,049 ; 2,664 ; 3,619 ; 4,235 ; \dots$$

$$\dots 5,190 ; 5,805 \}$$

$$(k) \sin 2x + \tan 2x = 0$$

$$\underline{CE}: 2x \neq \frac{\pi}{2} + k\pi$$

$$\sin 2x + \frac{\sin 2x}{\cos 2x} = 0$$

$$\Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$\Leftrightarrow \sin 2x \left(1 + \frac{1}{\cos 2x} \right) = 0 \Leftrightarrow \sin 2x \frac{(1 + \cos 2x)}{\cos 2x} = 0$$

for cause des CE

$$\bullet \sin 2x = 0 \Leftrightarrow 2x = k\pi$$

$$\Leftrightarrow x = \frac{k\pi}{2}$$

$$\bullet \cos 2x = -1 \Leftrightarrow 2x = \pi + 2k\pi$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\pi$$

$$S_g: \left\{ \frac{k\pi}{2} \mid k \in \mathbb{Z} \right\}$$

$$S_p: \left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$$

$$(l) \cos 2x = \cos x + 1$$

$$2 \cos^2 x - 1 - \cos x - 1 = 0 \Leftrightarrow 2 \cos^2 x - \cos x - 2 = 0$$

$$\Delta = 17$$

$$\cos x_{1,2} = \frac{1 \pm \sqrt{17}}{4} \begin{cases} \approx 1,281 \\ \approx -0,781 \end{cases}$$

$$\bullet \cos x \approx 1,281 \Rightarrow \text{imp}$$

$$\bullet \cos x \approx -0,781 \Leftrightarrow \begin{cases} x \approx 2,467 + 2k\pi \\ x \approx -2,467 + 2k\pi \end{cases}$$

$$S_g: \left\{ -2,467 + 2k\pi; 2,467 + 2k\pi \mid k \in \mathbb{Z} \right\}$$

$$S_p: \left\{ 2,467; 5,609 \right\}$$

$$(m) 3 \cos x (\sin x + \tan x) = 2(1 + \cos x)$$

$$CE : x \neq \frac{\pi}{2} + k\pi$$

$$3 \cos x \left(\sin x + \frac{\sin x}{\cos x} \right) = 2(1 + \cos x)$$

$$\Leftrightarrow 3 \sin x \cos x \left(1 + \frac{1}{\cos x} \right) = 2(1 + \cos x)$$

$$\Leftrightarrow 3 \sin x \cancel{\cos x} \frac{(1 + \cos x)}{\cancel{\cos x}} = 2(1 + \cos x)$$

cf CE

$$\Leftrightarrow 3 \sin x (1 + \cos x) = 2(1 + \cos x)$$

$$\Leftrightarrow 3 \sin x (1 + \cos x) - 2(1 + \cos x) = 0$$

$$\Leftrightarrow (1 + \cos x)(3 \sin x - 2) = 0$$

⊗ suite : voir page suivante

$$(n) 5 \sin^2 x - 2 \cos^2 x - 3 \sin x \cos x = 0$$

$$\Leftrightarrow \frac{5 \sin^2 x - 2 \cos^2 x - 3 \sin x \cos x}{\cos^2 x} = 0 \quad CE : x \neq \frac{\pi}{2} + k\pi$$

$$\Leftrightarrow 5 \tan^2 x - 2 - 3 \tan x = 0$$

$$\Delta = 9 + 40 = 49 \quad \tan x_{1,2} = \frac{3 \pm 7}{10} \begin{cases} 1 \\ -\frac{2}{5} \end{cases}$$

$$\tan x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi$$

$$\tan x = -\frac{2}{5} \Leftrightarrow x = -0,381 + k\pi$$

$$S_g : \left\{ -0,381 + k\pi ; \frac{\pi}{4} + k\pi \right\} k \in \mathbb{Z}$$

$$S_p : \left\{ \frac{\pi}{4} ; 2,761 ; \frac{3\pi}{4} ; 5,302 \right\}$$

$$(*) \cdot 1 + \cos x = 0 \Leftrightarrow \cos x = -1 \Leftrightarrow x = \pi + 2k\pi$$

$$\cdot 3 \sin x - 2 = 0 \Leftrightarrow \sin x = \frac{2}{3}$$

$$\Leftrightarrow \begin{cases} x = 0,73 + 2k\pi \\ x = 2,41 + 2k\pi \end{cases}$$

$$S_g : \{ 0,73 + 2k\pi; 2,41 + 2k\pi; \pi + 2k\pi \mid k \in \mathbb{Z} \}$$

$$S_p : \{ 0,73; 2,41; \pi \}$$

$$(o) \sin x + \cos x = 1$$

On utilise les formules on $\tan \frac{x}{2} = t$

$$\Leftrightarrow \left(\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1 \right) (1+t^2)$$

$$\Leftrightarrow 2t + 1 - t^2 = 1 + t^2 \Leftrightarrow 2t - 2t^2 = 0 \Leftrightarrow 2t(1-t) = 0$$

$$\cdot \tan \frac{x}{2} = 0 \Leftrightarrow \frac{x}{2} = k\pi \Leftrightarrow x = 2k\pi$$

$$\cdot \tan \frac{x}{2} = 1 \Leftrightarrow \frac{x}{2} = \frac{\pi}{4} + k\pi \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$$

$$S_g: \left\{ \frac{\pi}{2} + 2k\pi, 2k\pi \right\} k \in \mathbb{Z}$$

$$S_p: \left\{ 0, \frac{\pi}{2} \right\}$$

$$(p) \cos x + \sqrt{3} \sin x = 1$$

$$t = \tan \frac{x}{2}$$

$$\Leftrightarrow \left(\frac{1-t^2}{1+t^2} + \sqrt{3} \frac{2t}{1+t^2} = 1 \right) \cdot (1+t^2)$$

$$\Leftrightarrow 1 - t^2 + 2\sqrt{3}t = 1 + t^2 \Leftrightarrow 2\sqrt{3}t - 2t^2 = 0$$

$$\Leftrightarrow 2t(\sqrt{3} - t) = 0$$

$$\cdot \tan \frac{x}{2} = 0 \Leftrightarrow \frac{x}{2} = k\pi \Leftrightarrow x = 2k\pi$$

$$\cdot \tan \frac{x}{2} = \sqrt{3} \Leftrightarrow \frac{x}{2} = \frac{\pi}{3} + k\pi \Leftrightarrow x = \frac{2\pi}{3} + 2k\pi$$

$$S_g: \left\{ \frac{2\pi}{3} + 2k\pi, 2k\pi \right\} k \in \mathbb{Z}$$

$$S_p: \left\{ 0, \frac{2\pi}{3} \right\}$$

$$(q) \sqrt{2} \cos x - \sqrt{2} \sin x = 1$$

$$t = \tan \frac{x}{2}$$

$$\Rightarrow \sqrt{2} \frac{1-t^2}{1+t^2} - \sqrt{2} \frac{2t}{1+t^2} = 1$$

$$\Rightarrow \sqrt{2} (1-t^2) - 2\sqrt{2}t = 1+t^2$$

$$\Rightarrow \sqrt{2} - \sqrt{2}t^2 - 2\sqrt{2}t - 1 - t^2 = 0 \Rightarrow (-\sqrt{2}-1)t^2 - 2\sqrt{2}t - 1+\sqrt{2} = 0$$

$$\Delta = 8 - 4(-\sqrt{2}-1)(\sqrt{2}-1) = 8 + 4(\sqrt{2}+1)(\sqrt{2}-1) \\ = 8 + 4(1) = 12$$

$$\tan \frac{x_{1,2}}{2} = \frac{-2\sqrt{2} \pm 2\sqrt{3}}{-2(\sqrt{2}+1)} = \frac{-\sqrt{2} \pm \sqrt{3}}{\sqrt{2}+1}$$

$$\tan \frac{x_1}{2} = \frac{-\sqrt{2} + \sqrt{3}}{\sqrt{2}+1} \Leftrightarrow \frac{x_1}{2} = \frac{\pi}{24} + k\pi$$

$$\Leftrightarrow x = \frac{\pi}{12} + 2k\pi$$

(*) milé page suivante

$$(r) 2 \sin x - 3 \cos x = 3$$

$$t = \tan \frac{x}{2}$$

$$\Rightarrow \frac{4t}{1+t^2} - 3 \frac{1-t^2}{1+t^2} = 3$$

$$\Rightarrow 4t - 3 + 3t^2 = 3 + 3t^2 \Leftrightarrow 4t - 6 = 0$$

$$\Rightarrow t = \frac{3}{2}$$

$$\tan \frac{x}{2} = \frac{3}{2} \Leftrightarrow \frac{x}{2} = 0,983 + k\pi \Leftrightarrow x = 1,966 + 2k\pi$$

$$S_g : \{ 1,966 + 2k\pi \mid k \in \mathbb{Z} \}$$

$$S_p : \{ 1,966 \}$$

$$\textcircled{3} \quad \tan \frac{x_2}{2} = \frac{-\sqrt{2}-\sqrt{3}}{\sqrt{2}+1} \Leftrightarrow \frac{x_2}{2} = -\frac{7\pi}{24} + 2k\pi$$

$$\Leftrightarrow x_2 = -\frac{7\pi}{12} + 2k\pi$$

$$S_g : \left\{ -\frac{7\pi}{12} + 2k\pi, \frac{\pi}{12} + 2k\pi \right\} k \in \mathbb{Z}$$

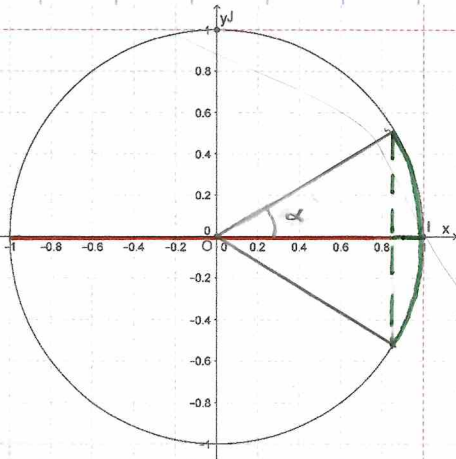
$$S_p : \left\{ \frac{\pi}{12}, \frac{17\pi}{12} \right\}$$

Inéquations trigonométriques : Solutions

0.1 Inéquations de base

Résoudre les inéquations suivantes et exprimer les solutions sur l'intervalle $[-\pi, \pi]$:

1. $\cos x > \frac{\sqrt{3}}{2}$



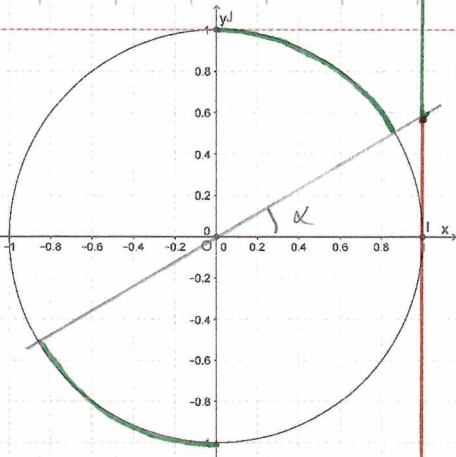
$$x = 30^\circ \left(\frac{\pi}{6} \right)$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{ \left] 2k\pi, \frac{\pi}{6} + 2k\pi \right[\cup \left] \frac{11\pi}{6} + 2k\pi, (k+1)2\pi \right[\right\}$$

$$S_p: \left] 0, \frac{\pi}{6} \right[\cup \left] \frac{11\pi}{6}, 2\pi \right[$$

2. $3 \tan y - \sqrt{3} \geq 0$

$$\Rightarrow \tan y \geq \frac{\sqrt{3}}{3}$$

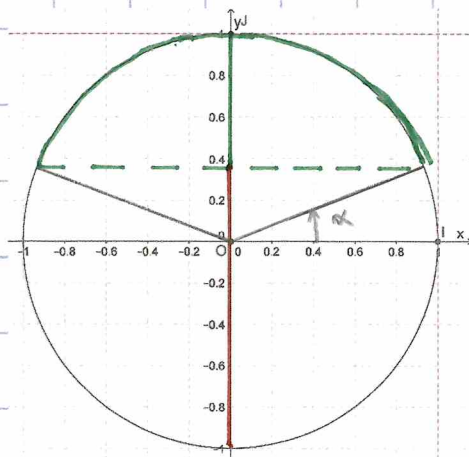


$$x = \frac{\pi}{6}$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{ \left[\frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi \right[\right\}$$

$$S_p: \left[\frac{\pi}{6}, \frac{\pi}{2} \right[\cup \left[\frac{7\pi}{6}, \frac{3\pi}{2} \right[$$

$$3. 1 - 3 \sin \left(x - \frac{\pi}{4} \right) \leq 0$$



$$\Leftrightarrow \sin \left(x - \frac{\pi}{4} \right) \geq \frac{1}{3}$$

$$\alpha = \sin^{-1} \left(\frac{1}{3} \right) \approx 0,34$$

$$\Rightarrow 0,34 \leq x - \frac{\pi}{4} \leq 2,80$$

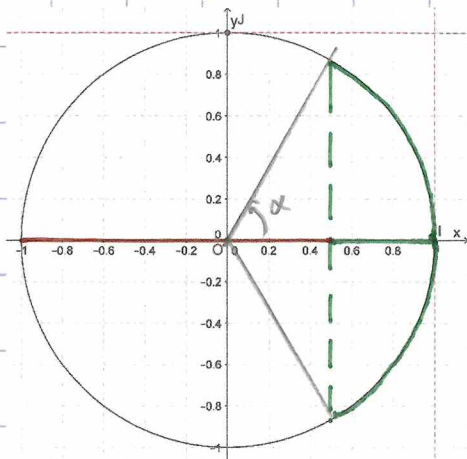
$$\Leftrightarrow 1,125 \leq x \leq 3,586$$

$$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ [1,125 + 2k\pi; 3,586 + 2k\pi] \right\}$$

$$S_p : [1,125; 3,586]$$

$$4. 1 - 2 \cos 5y < 0$$

$$\Leftrightarrow \cos 5y > \frac{1}{2}$$



$$\alpha = \frac{\pi}{3}$$

$$5y \in \left[0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right] + 2k\pi$$

$$\Leftrightarrow y \in \left[0, \frac{\pi}{15}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] + \frac{2k\pi}{5}$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{ \left[\frac{2k\pi}{5}, \frac{\pi}{15} + \frac{2k\pi}{5} \right] \cup \left[\frac{2\pi}{3} + \frac{2k\pi}{5}, \frac{4\pi}{3} + \frac{2k\pi}{5} \right] \right\}$$

$$S_p: \left[0, \frac{\pi}{15}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] \cup \left[\frac{2\pi}{5}, \frac{7\pi}{15}\right] \dots$$

$$\dots \cup \left[\frac{11\pi}{15}, \frac{4\pi}{5}\right] \cup \left[\frac{4\pi}{5}, \frac{13\pi}{15}\right] \cup \left[\frac{17\pi}{15}, \frac{6\pi}{5}\right] \dots$$

$$\dots \cup \left[\frac{6\pi}{5}, \frac{19\pi}{15}\right] \cup \left[\frac{23\pi}{15}, \frac{8\pi}{5}\right] \cup \left[\frac{8\pi}{5}, \frac{5\pi}{3}\right] \dots$$

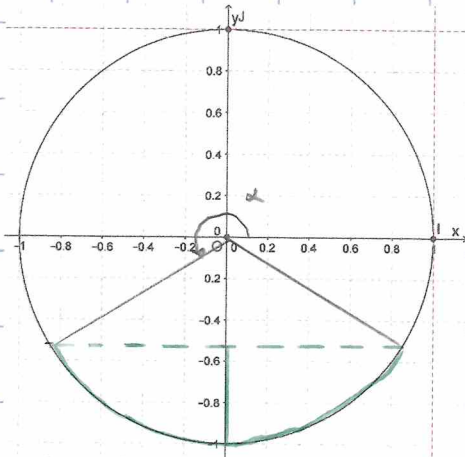
$$\dots \cup \left[\frac{23\pi}{5}, 2\pi\right]$$

$$\text{ou } S_p: \left[0, \frac{\pi}{15}\right] \cup \left[\frac{2\pi}{3}, \frac{7\pi}{15}\right] \cup \left[\frac{11\pi}{15}, \frac{13\pi}{5}\right] \dots$$

$$\dots \cup \left[\frac{17\pi}{15}, \frac{19\pi}{15}\right] \cup \left[\frac{23\pi}{5}, \frac{5\pi}{3}\right] \cup \left[\frac{25\pi}{5}, 2\pi\right]$$

$$5. 2 \sin\left(3t - \frac{\pi}{5}\right) + 1 < 0$$

$$\Leftrightarrow \sin\left(3t - \frac{\pi}{5}\right) < -\frac{1}{2}$$



$$\alpha = \frac{7\pi}{6}$$

$$\Rightarrow \frac{7\pi}{6} < 3t - \frac{\pi}{5} < \frac{11\pi}{6}$$

$$\Leftrightarrow \frac{7\pi}{6} + \frac{\pi}{5} < 3t < \frac{11\pi}{6} + \frac{\pi}{5}$$

$$\Leftrightarrow \frac{41\pi}{30} < 3t < \frac{61\pi}{30} + 2k\pi$$

$$\Leftrightarrow \frac{41\pi}{90} < t < \frac{61\pi}{90} + \frac{2k\pi}{3}$$

$$S_g : \bigcup_{k \in \mathbb{Z}} \left] \frac{41\pi}{90} + \frac{2k\pi}{3} ; \frac{61\pi}{90} + \frac{2k\pi}{3} \right[\quad \left(\frac{2\pi}{3} = \frac{60\pi}{90} \right)$$

$$S_p : \left] \frac{41\pi}{90} , \frac{61\pi}{90} \right[\cup \left] \frac{101\pi}{90} , \frac{121\pi}{90} \right[\dots$$

$$\dots \cup \left] \frac{161\pi}{90} , \frac{181\pi}{90} \right[$$

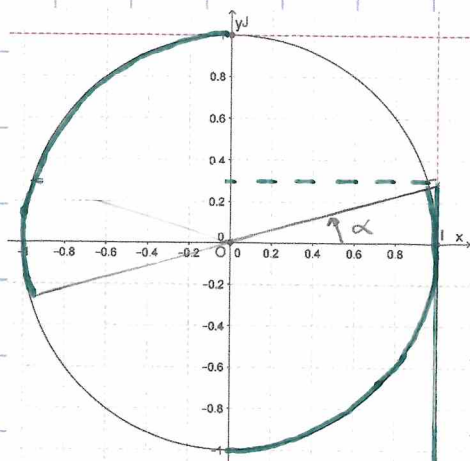
$\frac{181\pi}{90} - 2\pi = \frac{\pi}{90}$

$$: \left[0 , \frac{\pi}{90} \right[\cup \left] \frac{41\pi}{90} , \frac{61\pi}{90} \right[\dots$$

$$\dots \cup \left] \frac{101\pi}{90} , \frac{121\pi}{90} \right[\cup \left] \frac{161\pi}{90} , 2\pi \right[$$

$$6. 3 \tan 3x - 1 \leq 0$$

$$\Rightarrow \tan 3x \leq \frac{1}{3}$$



$$\kappa = \tan^{-1}\left(\frac{1}{3}\right) \approx 0,32$$

$$3x \in [0; 0,32] \cup \left] \frac{\pi}{2}, \pi \right] + k\pi$$

$$x \in [0; 0,107] \cup \left] \frac{\pi}{6}, \frac{\pi}{3} \right] + \frac{k\pi}{3}$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{ \left[\frac{k\pi}{3}; 0,107 + \frac{k\pi}{3} \right] \cup \left[\frac{\pi}{6} + \frac{k\pi}{3}, \frac{(k+1)\pi}{3} \right] \right\}$$

$$S_p: k=0 \quad S_0: [0; 0,107] \cup \left] \frac{\pi}{6}, \frac{\pi}{3} \right]$$

$$k=1 \quad S_1: \left[\frac{\pi}{3}; 1,154 \right] \cup \left] \frac{\pi}{2}, \frac{2\pi}{3} \right]$$

$$k=2 \quad S_2: \left[\frac{2\pi}{3}; 2,202 \right] \cup \left] \frac{5\pi}{6}, \pi \right]$$

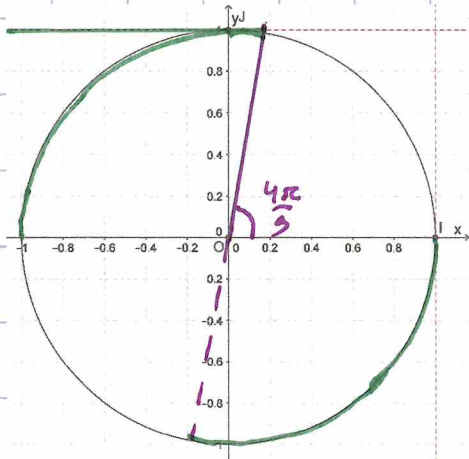
$$k=3 \quad S_3: [\pi; 3,249] \cup \left] \frac{7\pi}{6}, \frac{4\pi}{3} \right]$$

$$k=4 \quad S_4: \left[\frac{4\pi}{3}; 4,296 \right] \cup \left] \frac{3\pi}{2}, \frac{5\pi}{3} \right]$$

$$k=5 \quad S_5: \left[\frac{5\pi}{3}; 5,343 \right] \cup \left] \frac{11\pi}{6}, 2\pi \right]$$

$$S_p = S_0 \cup S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$$

$$7. \cot \frac{4\pi}{9} - \cot \left(3x - \frac{\pi}{3} \right) \geq 0 \quad (\Rightarrow) \cot \left(3x - \frac{\pi}{3} \right) \leq \cot \frac{4\pi}{9}$$



$$\frac{4\pi}{9} \leq 3x - \frac{\pi}{3} \leq \pi + 2k\pi$$

$$\Rightarrow \frac{4\pi}{9} + \frac{\pi}{3} \leq 3x \leq \pi + \frac{\pi}{3} + 2k\pi$$

$$\Leftrightarrow \frac{7\pi}{9} \leq 3x \leq \frac{4\pi}{3} + 2k\pi$$

$$\Leftrightarrow \frac{7\pi}{27} \leq x \leq \frac{4\pi}{9} + \frac{2k\pi}{3}$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{ \left[\frac{7\pi}{27} + \frac{2k\pi}{3}, \frac{4\pi}{9} + \frac{2k\pi}{3} \right] \right\}$$

$$\frac{3\pi}{27} / \frac{3\pi}{9}$$

$$S_g: \quad k=0: \left[\frac{7\pi}{27}, \frac{4\pi}{9} \right] \quad \left| \quad k=1: \left[\frac{16\pi}{27}, \frac{7\pi}{9} \right] \right.$$

$$k=2: \left[\frac{25\pi}{27}, \frac{10\pi}{9} \right] \quad \left| \quad k=3: \left[\frac{34\pi}{27}, \frac{13\pi}{9} \right] \right.$$

$$k=4: \left[\frac{43\pi}{27}, \frac{16\pi}{9} \right] \quad \left| \quad k=5: \left[\frac{52\pi}{27}, \frac{19\pi}{9} \right] \right.$$

$$\otimes > 2\pi$$

$$\Rightarrow \left[\frac{52\pi}{27}, 2\pi \right] \cup \left[0, \frac{\pi}{9} \right]$$

$$\Rightarrow S_g: \left[0, \frac{\pi}{9} \right] \cup \left[\frac{7\pi}{27}, \frac{4\pi}{9} \right] \cup \left[\frac{16\pi}{27}, \frac{7\pi}{9} \right] \cup \left[\frac{25\pi}{27}, \frac{10\pi}{9} \right] \dots$$

$$\dots \cup \left[\frac{34\pi}{27}, \frac{13\pi}{9} \right] \cup \left[\frac{43\pi}{27}, \frac{16\pi}{9} \right] \cup \left[\frac{52\pi}{27}, 2\pi \right]$$

0.2 Inéquations

Résoudre les inéquations suivantes et exprimer les solutions sur l'intervalle $[0, 2\pi[$: ~~$[-\pi, \pi]$~~

$$1. \sin^2 3x \leq \frac{3}{4} \quad (\Leftrightarrow \sin^2 3x - \frac{3}{4} \leq 0)$$

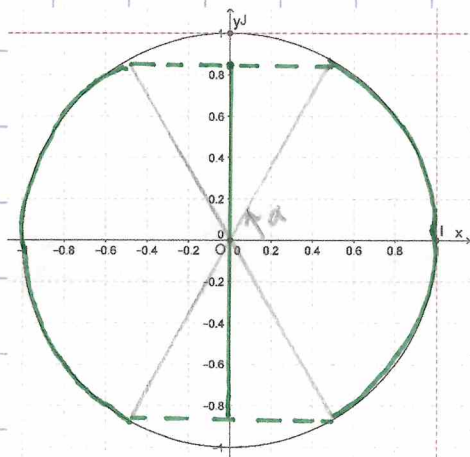
$$\text{On pose } y = \sin 3x \Rightarrow y^2 - \frac{3}{4} \leq 0$$

y		$-\frac{\sqrt{3}}{2}$		$\frac{\sqrt{3}}{2}$		
In		+	0	-	0	+

$$\Rightarrow -\frac{\sqrt{3}}{2} \leq y \leq \frac{\sqrt{3}}{2}$$

$$\Leftrightarrow -\frac{\sqrt{3}}{2} \leq \sin 3x \leq \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}$$



$$3x \in [0, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, \frac{4\pi}{3}] \dots$$

$$\dots \cup [\frac{5\pi}{3}, 2\pi[+ 2k\pi$$

$$\Leftrightarrow x \in [0, \frac{\pi}{9}] \cup [\frac{2\pi}{9}, \frac{4\pi}{9}] \cup [\frac{5\pi}{9}, \frac{2\pi}{3}] + \frac{2k\pi}{3}$$

$$S_g : \bigcup_{k \in \mathbb{Z}} \left\{ \left[\frac{2k\pi}{3}, \frac{\pi}{9} + \frac{2k\pi}{3} \right] \cup \left[\frac{2\pi}{9} + \frac{2k\pi}{3}, \frac{4\pi}{9} + \frac{2k\pi}{3} \right] \dots \right. \\ \left. \dots \cup \left[\frac{5\pi}{9} + \frac{2k\pi}{3}, \frac{(k+1)2\pi}{3} \right] \right\} \quad \frac{6\pi}{3}$$

$$S_p: k=0 \quad \left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}, \frac{2\pi}{3}\right] = S_1$$

$$k=1 \quad \left[\frac{4\pi}{3}, \frac{2\pi}{3}\right] \cup \left[\frac{8\pi}{9}, \frac{10\pi}{9}\right] \cup \left[\frac{11\pi}{9}, \frac{4\pi}{3}\right] = S_2$$

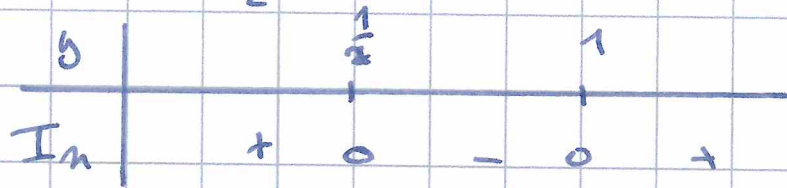
$$k=2 \quad \left[\frac{4\pi}{3}, \frac{13\pi}{9}\right] \cup \left[\frac{14\pi}{9}, \frac{16\pi}{9}\right] \cup \left[\frac{17\pi}{9}, 2\pi\right] = S_3$$

$$S_p = S_1 \cup S_2 \cup S_3$$

$$2. 2\sin^2 x - 3\sin x + 1 < 0$$

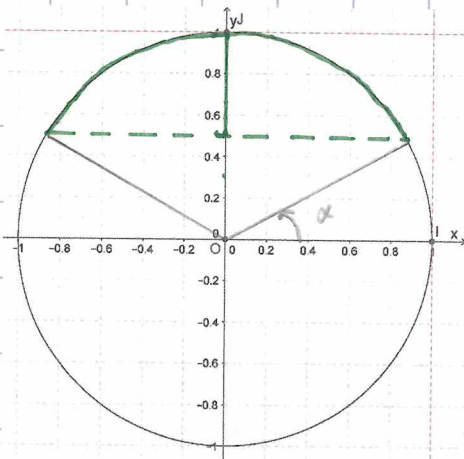
$$\Delta = 9 - 8 = 1$$

$$y_{1,2} = \frac{3 \pm 1}{4} < \frac{1}{2}$$



$$\rightarrow y \in]\frac{1}{2}, 1[$$

$$\Leftrightarrow \sin x \in]\frac{1}{2}, 1[$$



$$\alpha = \frac{\pi}{6}$$

$$x \in]\frac{\pi}{6}, \frac{\pi}{2}[\cup]\frac{5\pi}{2}, \frac{5\pi}{6}[+ 2k\pi$$

$$\Leftrightarrow x \in]\frac{\pi}{6}; \frac{5\pi}{6}[\setminus \left\{ \frac{\pi}{2} \right\} + 2k\pi$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{]\frac{\pi}{6} + 2k\pi; \frac{5\pi}{6} + 2k\pi[\setminus \left\{ \frac{\pi}{2} + 2k\pi \right\} \right\}$$

$$S_p:]\frac{\pi}{6}, \frac{\pi}{2}[\cup]\frac{\pi}{2}, \frac{5\pi}{6}[$$

$$3. 3 \sin x + 2 \cos x > 1$$

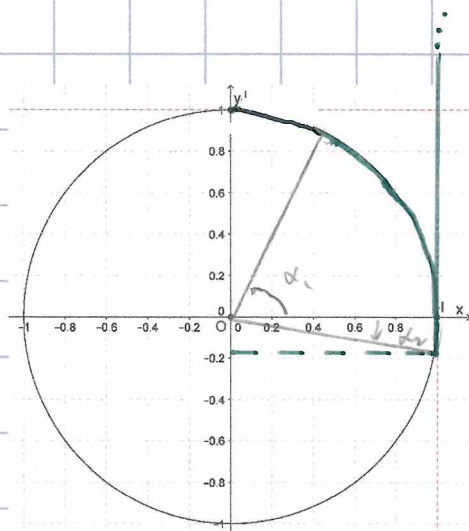
On pose $t = \tan \frac{x}{2}$. L'éq devient

$$\left(3 \cdot \frac{2t}{1+t^2} + 2 \cdot \frac{1-t^2}{1+t^2} > 1 \right) (1, t^2) \quad (cos > 0)$$

$$\Leftrightarrow 6t + 2 - 2t^2 > 1 + t^2 \Leftrightarrow -3t^2 + 6t + 1 > 0$$

$$\Delta = 36 + 12 = 48 \quad \text{et} \quad t_{1,2} = \frac{-6 \pm 4\sqrt{3}}{-6} = 1 \pm \frac{2\sqrt{3}}{3} \begin{cases} \approx 2,15 \\ \approx -0,15 \end{cases}$$

t	t_2	t_1			
I_n	-	0	+	0	-



$$t \in]t_2, t_1[$$

$$\alpha_1 \approx 1,136 \quad \text{et} \quad \alpha_2 \approx -0,153$$

$$\Rightarrow \frac{x}{2} \in]-0,153; 1,136[+ k\pi$$

$$\Leftrightarrow x \in]-0,307; 2,273[+ 2k\pi$$

$$S_g : \bigcup_{k \in \mathbb{Z}} \left\{]2k\pi; 2,273 + 2k\pi[\cup]5,977 + 2k\pi; (k+1)2\pi[\right\}$$

$$S_p : [0; 2,273[\cup]5,977; 2\pi[$$

$$4. \frac{\cos 2x}{1 - 2 \sin 2x} \leq 0$$

C'est une ineq générale $\rightarrow$ T.S.

On cherche les zéros.

$$\underline{N}: \cos 2x = 0 \Leftrightarrow 2x = \frac{\pi}{2} + k\pi$$

$$\Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$\Rightarrow \text{sur } [0, 2\pi[: \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\underline{D}: 1 - 2 \sin 2x = 0 \Leftrightarrow \sin 2x = \frac{1}{2}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 2x = \frac{\pi}{6} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{12} + k\pi \end{cases}$$

$$\Rightarrow \text{sur } [0, 2\pi[: \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{4}$	$\frac{5\pi}{12}$	$\frac{3\pi}{4}$	$\frac{13\pi}{12}$	$\frac{5\pi}{4}$	$\frac{17\pi}{12}$	$\frac{7\pi}{4}$	2π
$\cos 2x$	+	+	0	-	-	+	+	-	-	+
$1 - 2 \sin 2x$	+	0	-	0	+	+	-	-	0	+
I_n	+	+	-	+	-	+	-	+	-	+

$$S_p :]\frac{\pi}{12}, \frac{\pi}{4}] \cup]\frac{5\pi}{12}, \frac{3\pi}{4}] \cup]\frac{13\pi}{12}, \frac{5\pi}{4}] \cup]\frac{17\pi}{12}, \frac{7\pi}{4}]$$

$$S_g = \bigcup_{k \in \mathbb{Z}} \left\{ S_p + 2k\pi \right\}$$

$$\textcircled{*} \frac{\pi}{2} \in \left[\frac{\pi}{4}, \frac{3\pi}{4} \right] \text{ et } \cos 2 \frac{\pi}{2} = \cos \pi = -1 < 0$$

$$\textcircled{*} \frac{\pi}{2} \in \left[\frac{5\pi}{12}, \frac{13\pi}{12} \right] \text{ et } 1 - 2 \sin 2 \frac{\pi}{2} = 1 - 2 \sin \pi = 1 > 0$$



exercice diff $\Rightarrow$ rigueur et lentem

5. $1 + \cos x > \sin^2 x$

$$\Leftrightarrow 1 + \cos x > 1 - \cos^2 x \Leftrightarrow \cos^2 x + \cos x > 0$$

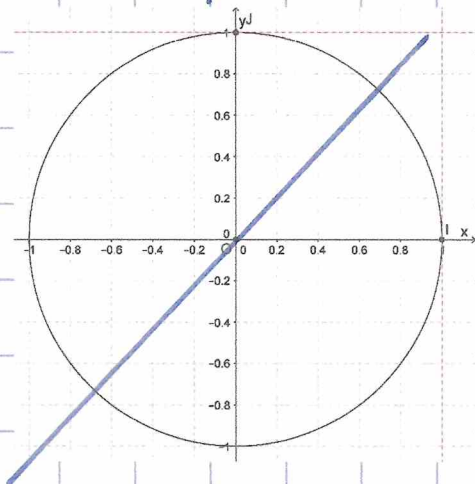
$$\Leftrightarrow \cos x (1 + \cos x) > 0$$

Zeige : • $\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi$

sur $[0, 2\pi[$: $\frac{\pi}{2}, \frac{3\pi}{2}$

• $1 + \cos x = 0 \Leftrightarrow \cos x = -1 \Leftrightarrow x = \pi + 2k\pi$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π		
$\cos x$	+	0	-	-	0	+	
$1 + \cos x$	+	+	0	+	+	+	
	+	0	-	0	-	0	+



$$S_p : [0, \frac{\pi}{2}[\cup] \frac{3\pi}{2}, 2\pi[$$

$$S_g = \bigcup_{k \in \mathbb{Z}} \{ S_p + 2k\pi \}$$

⊗ Rappel : c'est une des exceptions des T.S.

$$-1 \leq \cos x \leq 1 \Leftrightarrow -1 + 1 \leq \cos x + 1 \leq 1 + 1$$

$$\Leftrightarrow 0 \leq \cos x + 1 \leq 2 \Leftrightarrow \cos x + 1 \geq 0 !$$

(Idem pour $\cos x - 1$, $\sin x - 1$, $\sin x + 1$, $1 - \cos x$, $1 + \cos x$, $1 - \sin x$ et $1 + \sin x$)

$$6. \frac{2 \sin 2x - 1}{\cos 2x - 3 \cos x + 2} > 0$$

Pour faire le tableau de signe, on cherche les zéros

$$\underline{N}: 2 \sin 2x - 1 = 0 \Leftrightarrow \sin 2x = \frac{1}{2}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 2x = \frac{5\pi}{6} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{12} + k\pi \end{cases}$$

Sur $[0, 2\pi[$, les solutions sont: $\left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$

$$\underline{D}: \cos 2x - 3 \cos x + 2 = 0 \Leftrightarrow 2 \cos^2 x - 1 - 3 \cos x + 2 = 0$$

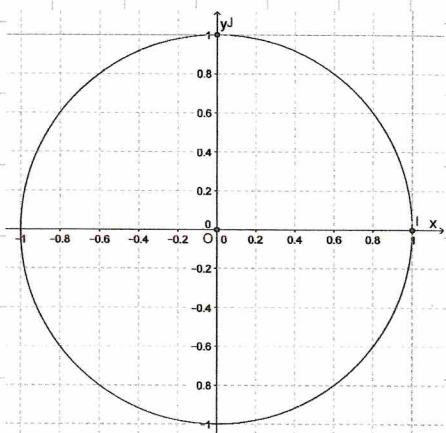
$$\Leftrightarrow 2 \cos^2 x - 3 \cos x + 1 = 0$$

$$\Delta = 1, \quad \cos x_{1,2} = \frac{3 \pm 1}{4} < \frac{1}{2}$$

$$\cos x = 1 \Leftrightarrow x = 2k\pi$$

$$\cos x = \frac{1}{2} \Leftrightarrow x = \pm \frac{\pi}{3} + 2k\pi$$

Rmq: D se factorise en $2 \left(\cos x - \frac{1}{2} \right) (\cos x - 1)$



Sur $[0, 2\pi[$, les solutions sont $\left\{ 0, \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{5\pi}{12}$	$\frac{13\pi}{12}$	$\frac{17\pi}{12}$	$\frac{5\pi}{3}$	2π
N	-	0	+	+	0	-	0	-
$2\cos x - 1$	+	+	0	-	-	-	-	+
$\cos x - 1$	0	-	-	-	-	-	-	-
I_n	+	+	0	-	+	+	0	-

$$S_p:]0, \frac{\pi}{12} [\cup] \frac{\pi}{3}, \frac{5\pi}{12} [\cup] \frac{13\pi}{12}, \frac{17\pi}{12} [\cup] \frac{5\pi}{3}, 2\pi [$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \{ S_p + 2k\pi \}$$

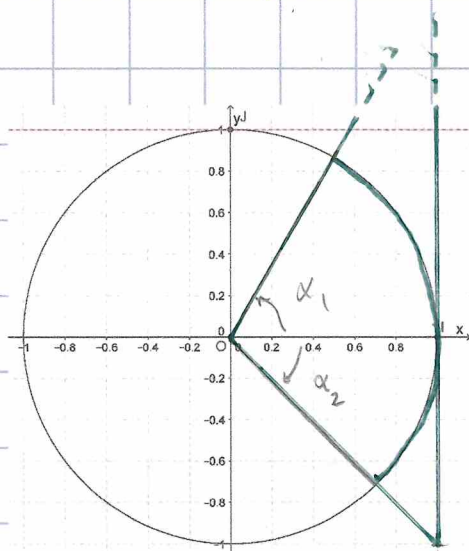
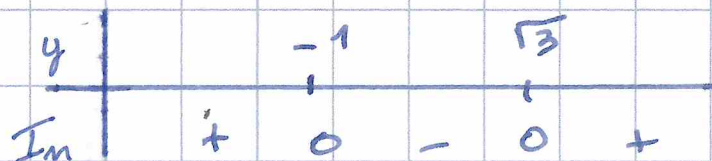
$$7. \tan^2 2x + (1 - \sqrt{3}) \tan 2x - \sqrt{3} \leq 0$$

$$\text{On pose } y = \tan 2x$$

$$y^2 + (1 - \sqrt{3})y - \sqrt{3} \leq 0$$

$$\begin{aligned} \Delta &= (1 - \sqrt{3})^2 + 4\sqrt{3} \\ &= 1 + 3 - 2\sqrt{3} + 4\sqrt{3} \\ &= 1 + 3 + 2\sqrt{3} \\ &= (1 + \sqrt{3})^2 \end{aligned}$$

$$y_{1,2} = \frac{-1 + \sqrt{3} \pm (1 + \sqrt{3})}{2} \begin{matrix} \sqrt{3} \\ -1 \end{matrix}$$



$$\begin{aligned} y &\in [-1, \sqrt{3}] \\ \tan 2x &\in [-1, \sqrt{3}] \end{aligned}$$

$$\alpha_1 = \frac{\pi}{3} \text{ et } \alpha_2 = -\frac{\pi}{4} = \frac{7\pi}{4}$$

$$2x \in [0, \frac{\pi}{3}] \cup [\frac{7\pi}{4}, 2\pi] + k\pi$$

$$\Leftrightarrow x \in [0, \frac{\pi}{6}] \cup [\frac{7\pi}{8}, \pi] + \frac{k\pi}{2}$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{ [0, \frac{\pi}{6}] \cup [\frac{7\pi}{8}, \pi] + \frac{k\pi}{2} \right\}$$

$$2x \in [-\frac{\pi}{4}, \frac{\pi}{3}] + k\pi \Leftrightarrow x \in [-\frac{\pi}{8}, \frac{\pi}{6}] + \frac{k\pi}{2}$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \left\{ [-\frac{\pi}{8}, \frac{\pi}{6}] + \frac{k\pi}{2} \cup [\frac{3\pi}{8} + \frac{k\pi}{2}, \frac{4\pi}{3} + \frac{k\pi}{2}] \right\}$$

$$S_p: k=0 \quad S_0: [0, \frac{\pi}{6}] \cup [\frac{5\pi}{8}, \pi]$$

$$k=1 \quad S_1: [\frac{\pi}{2}, \frac{2\pi}{3}] \cup [\frac{9\pi}{8}, \frac{3\pi}{2}]$$

$$k=2 \quad S_2: [\pi, \frac{7\pi}{6}] \cup [\frac{13\pi}{8}, 2\pi]$$

$$k=3 \quad S_3: [\frac{3\pi}{2}, \frac{5\pi}{3}] \cup [\frac{17\pi}{8}, \frac{5\pi}{2}]$$

$\underbrace{\hspace{10em}}_{> 2\pi}$
 $\Rightarrow [\frac{\pi}{8}, \frac{\pi}{2}]$
 to be modified (X)

$$S_p: S_0 \cup S_1 \cup S_2 \cup S_3$$

$$S_p: k=0 \quad S_0: [-\frac{\pi}{8}, \frac{\pi}{6}] \cup [0, \frac{\pi}{6}] \cup [\frac{15\pi}{8}, \pi]$$

$$k=1 \quad S_1: [\frac{3\pi}{8}, \frac{2\pi}{3}]$$

$$k=2 \quad S_2: [\frac{7\pi}{8}, \frac{7\pi}{6}]$$

$$k=3 \quad S_3: [\frac{11\pi}{8}, \frac{5\pi}{3}]$$

$$S = S_0 \cup S_1 \cup S_2 \cup S_3$$

8. $\sin 3x - \sin 2x + \sin x \geq 0$

$\otimes$ Simpson

$$\Leftrightarrow 2 \sin 2x \cos x - \sin 2x \geq 0$$

$$\Leftrightarrow \sin 2x (2 \cos x - 1) \geq 0$$

Zeilen: $\sin 2x = 0 \Leftrightarrow 2x = k\pi \Leftrightarrow x = \frac{k\pi}{2}$

in $[0, 2\pi[$: $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$2 \cos x - 1 \leq 0 \Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm \frac{\pi}{3} + 2k\pi$

in $[0, 2\pi[$: $\frac{\pi}{3}, \frac{5\pi}{3}$

x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π		
$\sin 2x$	0	+	+	0	-	0	-	0	
$2 \cos x - 1$		+	0	-	-	-	0	+	
I_n	0	+	0	-	0	+	0	-	0

$$S_p: \left[0, \frac{\pi}{3}\right] \cup \left[\frac{\pi}{2}, \pi\right] \cup \left[\frac{3\pi}{2}, \frac{5\pi}{3}\right]$$

$$S_g: \bigcup_{k \in \mathbb{Z}} \{S_p + 2k\pi\}$$