Angles remarquables et associés : Solutions

1. Vérifier les égalités suivantes :

α	0	30°	45°	60°	90°	180°	270°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	0	$\pm\infty$
$\cot\alpha$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$\pm\infty$	0

TABLE 1 – Valeurs des nombres trigonométriques d'angles remarquables

(a) $\sin 60° \cos 30° - \sin 30° \cos 60° = \sin 30°$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \,/\, \frac{1}{2}$ ✓

(b) $2\cos^2 30° - 1 = 1 - 2\sin^2 30° = \cos 60°$

$2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = \frac{1}{2} \,/\, 1 - 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} \,/\, \frac{1}{2}$ ✓

(c) $\sin^2 30° + \sin^2 60° + \tan^2 45° = 4\cos 60°$

$\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right) + 1 = 2 \,/\, 4\cdot\frac{1}{2} = 2$ ✓

(d) $\cos^2\frac{\pi}{3} + \sin^2\frac{\pi}{6} - \sin\frac{\pi}{4}\cos\frac{\pi}{4} = 0$

$\frac{1}{4} + \frac{1}{4} - \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = \frac{1}{2} - \frac{1}{2} = 0$ ✓

(e) $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{3} = 1$

$\frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$ ✓

(f) $\tan^2\frac{\pi}{3} + 4\cos^2\frac{\pi}{4} - \frac{3}{\sin^2\frac{\pi}{3}} = 1$

$3 + 4\cdot\frac{1}{2} - \frac{3}{\frac{3}{4}} = 3 + 2 - 4 = 1$ ✓

Il suffit de lire les valeurs de le tableau

2. Calculer les nombres trigonométriques suivants [1].

(a) $\cos 120° = -\cos 60° = -\frac{1}{2}$

(b) $\sin 330° = \sin(-30°) = -\sin 30° = -\frac{1}{2}$

(c) $\sin 480° = \sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

(d) $\cos 1050° = \cos 330° = \cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$

(e) $\tan \frac{15\pi}{4} = \tan \frac{3\pi}{4} = \tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

(f) $\cot \frac{16\pi}{3} = \cot \frac{4\pi}{3} = \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

(g) $\cos\left(-\frac{11\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

(h) $\tan 870° = \tan 150° = -\tan 30° = -\frac{\sqrt{3}}{3}$

(i) $\cos \frac{7\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$

(j) $\sin\left(-\frac{3\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

3. Calculer

(a) $\sin 2x + \tan x$ si $x = \frac{7\pi}{6}$

$$\sin\frac{7\pi}{3} + \tan\frac{7\pi}{6} = \sin\frac{\pi}{3} + \tan\frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3} = \frac{5\sqrt{3}}{6}$$

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(b) $\cos^2 x + \sin x$ si $x = \dfrac{5\pi}{3}$

$$\cos^2 \frac{5\pi}{3} + \sin \frac{5\pi}{3} = \cos^2\left(-\frac{\pi}{3}\right) + \sin\left(-\frac{\pi}{3}\right)$$

$$= \cos^2\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right) = \frac{1}{4} - \frac{\sqrt{3}}{2} = \frac{1-2\sqrt{3}}{4}$$

(c) $\cos x + \sin^2 x$ si $x = \dfrac{7\pi}{4}$

$$\cos \frac{7\pi}{4} + \sin^2 \frac{7\pi}{4} = \cos\left(-\frac{\pi}{4}\right) + \sin^2\left(-\frac{\pi}{4}\right)$$

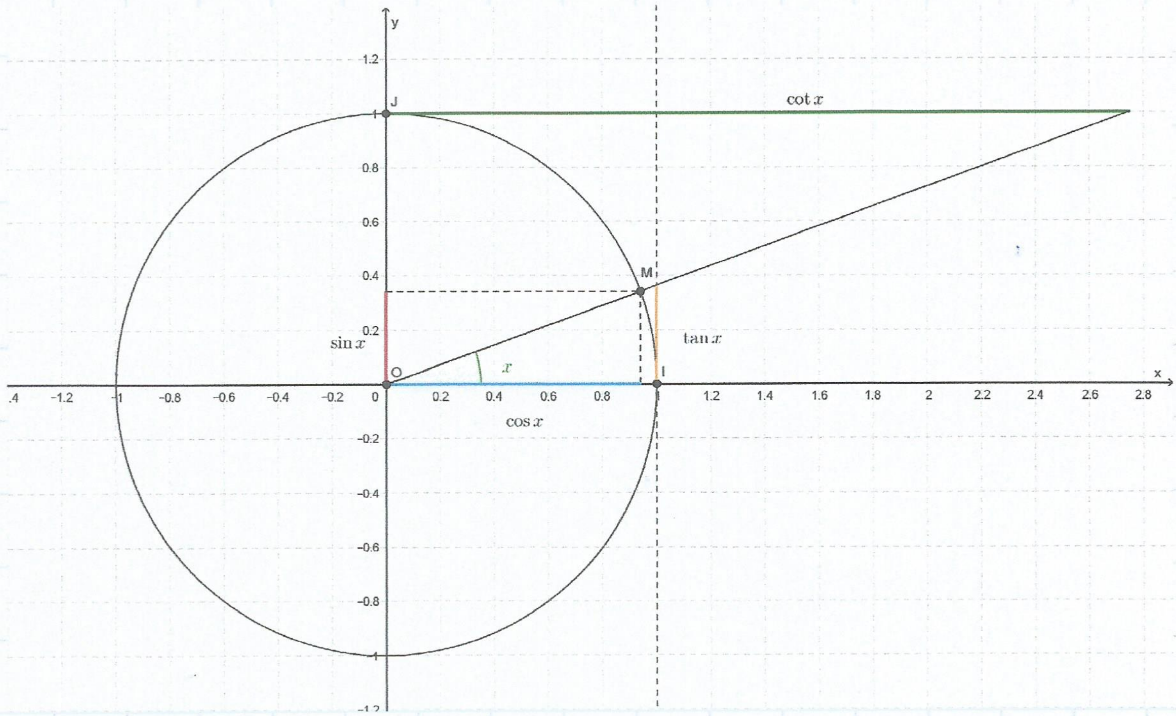
$$= \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2}+1}{2}$$

(d) $\cot x + \sin 2x$ si $x = -\dfrac{7\pi}{3}$

$$\cot\left(-\frac{7\pi}{3}\right) + \sin\left(-\frac{14\pi}{3}\right) = \cot\left(-\frac{\pi}{3}\right) + \sin\left(-\frac{2\pi}{3}\right)$$

$$= -\cot \frac{\pi}{3} - \sin \frac{\pi}{3} = -\frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{2} = -\frac{5\sqrt{3}}{6}$$

4. Simplifier les expressions suivantes



(a) $\sin(a - 90°)$ $= -\cos a$

(b) $\tan(a - 90°)$ $= -\cot a$

(c) $\sin(270° - a)$ $= -\cos a$

(d) $\tan(270° + a)$ $= -\cot a$

(e) $\sin(a - 540°)$ $= -\sin a$

(f) $\cos(7\pi - a)$ $= -\cos a$

(g) $\cot(a+3\pi)$ $= \cot a$

(h) $\cos(a+5\pi)+2\sin\left(\frac{\pi}{2}+a\right)+\cos(\pi-a)$

$= -\cos a + 2\cos a - \cos a$

$= 0$

(i) $\tan(x+3\pi)+\cot x+\tan\left(x-\frac{\pi}{2}\right)$

$= \tan x + \cot x - \cot x$

$= \tan x$

(j) $3\tan\left(x+\frac{3\pi}{2}\right)+2\tan\left(x-\frac{5\pi}{2}\right)-3\cot(x+\pi)$

$= -3\cot x - 2\cot x - 3\cot x$

$= -8\cot x$

(k) $$\frac{\sin\left(\frac{\pi}{2}-x\right)\sin(\pi+x)}{\cos\left(\frac{\pi}{2}+x\right)\cos(\pi-x)}+\frac{\sin(9\pi-x)\tan\left(x-\frac{\pi}{2}\right)}{\cot(x+5\pi)\cos\left(\frac{3\pi}{2}-x\right)}$$

$$= \frac{\cancel{\cos x}\,(-\cancel{\sin x})}{-\cancel{\sin x}\,(-\cancel{\cos x})} + \frac{\cancel{\sin x}\,(-\cancel{\cot x})}{\cancel{\cot x}\,(-\cancel{\sin x})}$$

$$= -1 + 1$$

$$= 0$$

(l) $$\frac{\sin\left(\frac{3\pi}{2}-x\right)\cos\left(x-\frac{\pi}{2}\right)\cot(x-2\pi)}{\tan(3\pi+x)\cot(-x)\cos(\pi+x)}$$

$$= \frac{-\cancel{\cos x}\ \sin x\ \cot x}{\underbrace{\tan x \cdot (-\cot x)}_{-1}\ (-\cancel{\cos x})}$$

$$= -\sin x \cdot \frac{\cos x}{\sin x}$$

$$= -\cos x$$