

Exercices de renforcement : Solutions

1. Effectuer et simplifier la réponse au maximum :

$$(a) [a + 9 - (3 + 2a)] - (5 - a + 3) - [2a - (4 - 5a)] =$$

$$= a + 9 - 3 - 2a - 5 + a - 3 - 2a + 4 - 5a = -7a + 2$$

$$(b) -9x^2 \cdot 8x^3 = -72x^5$$

$$(c) -x^2 \cdot (2x^4 - 3x^2 + 9x - 5) = -2x^6 + 3x^4 - 9x^3 + 5x^2$$

$$(d) (4x^8) \cdot (-3x^3) \cdot (-2x^5)^2 = -48x^{21}$$

$$(e) 2a(a-1) - 3a(a+1) = 2a^2 - 2a - 3a^2 - 3a$$
$$= -a^2 - 5a$$

$$(f) \cancel{2a(a-1)} - \cancel{3a(a+1)} = (-4x)(x^2y)(3xy^4)$$
$$= -12x^4y^5$$

$$(g) (2a+1)(3a-2) = 6a^2 - a - 2$$

$$(h) (2x^2+1)(3-2y) = 6x^2 - 2y - 4x^2y + 3$$

$$(i) \frac{30x^6y^5}{12x^2y^8} = \frac{5x^4}{2y^3}$$

$$(j) (2b - 5)(b + 2) - 3b(4 - 7b) = 2b^2 - b - 10 - 12b + 21b^2 \\ = 23b^2 - 13b - 10$$

$$(k) (5y - 1)[1 - 3(y + 2)] = (5y - 1)(-5 - 3y) \\ = -15y^2 - 22y + 5$$

$$(l) \frac{-4a^5 \cdot (-2a^4)^3}{28a^{10}} = \frac{32 a^{17}}{28 a^{10}} = \frac{8}{7} a^7$$

$$(m) 2a^6(a^2 + a - 1) - 3a^3(a^5 + 1) = 2a^8 + 2a^7 - 2a^6 - 3a^8 - 3a^3 \\ = -a^8 + 2a^7 - 2a^6 - 3a^3$$

$$(n) (2a^2 - ab + b^2) \cdot (a - 2b) = 2a^3 - 4a^2b - a^2b + 2ab^2 + ab^2 \dots \\ \dots - 2b^3 = 2a^3 - 5a^2b + 3ab^2 - 2b^3$$

$$(o) (3x^3 - 4)(2 - 5x) + (5 - x^2)(6 - x) = 6x^3 - 15x^4 + 20x - 8 \dots \\ \dots + x^3 - 6x^2 - 5x + 30 = -15x^4 + 7x^3 - 6x^2 + 15x + 22$$

$$(p) (1 - x^2) \cdot (2x + 3) \cdot 5x^4 = (-2x^3 - 3x^2 + 2x + 3)5x^4 \\ = -10x^7 - 15x^6 + 10x^5 + 15x^4$$

$$(q) \left(\frac{1}{2}ab - 1\right)(3ab + 2) = \frac{3}{2}a^2b^2 - 2ab - 2$$

$$(r) 2a^2 - 3b^2 + (a + b)(a - 2b) - (2a - b)(a + 3b) = 2a^2 - 3b^2 + a^2 - ab - 2b^2 \dots \\ \dots - (2a^2 + 5ab - 3b^2) = a^2 - 6ab - 2b^2$$

$$(s) 2x(x^2 - xy + y^2) - 2y(x^2 - 2y^2) = 2x^3 - 2x^2y + 2xy^2 - 2x^2y + 4y^3 \\ = 2x^3 - 4x^2y + 2xy^2 + 4y^3$$

$$(t) (x+1)(x-2)(3-x) - (x-1)(x+2)(x-3) = (x^2 - x - 2)(3-x) \dots$$

$$- (x^2 + x - 2)(x-3) = -x^3 + 4x^2 - x - 6 - x^3$$

$$+ 2x^2 + 5x - 6 = -2x^3 + 6x^2 + 4x - 12$$

2. Développer :

$$(a) (a^2 + 2b)^2 = a^4 + 4a^2b + 4b^2$$

$$(b) \left(\frac{x^3}{4} - \frac{y^2}{2}\right)^2 = \frac{x^6}{16} - \frac{x^3y^2}{4} + \frac{y^4}{4}$$

$$(c) (5 - x^3)(5 + x^3) = 25 - x^6$$

$$(d) (-2 - 3x)(-2 + 3x) = 4 - 9x^2$$

$$(e) \left(\frac{2a^2}{3b} - \frac{3b^3}{2a}\right)^2 = \frac{4a^4}{9b^2} - 2ab^2 + \frac{9b^6}{4a^2}$$

$$(f) (x+y)(x-y)(x^2+y^2) = (x^2 - y^2)(x^2 + y^2) = x^4 - y^4$$

$$(g) (2a + b - c)(2a + b + c) = [(2a + b) - c][(2a + b) + c]$$

$$= 4a^2 + 4ab + b^2 - c^2$$

$$(h) (4x - 3)^2 + (2 - 3x)(2 + 3x) = 16x^2 - 24x + 9 + 4 - 9x^2$$

$$= 7x^2 - 24x + 13$$

$$(i) 2x \cdot (5x - 3)^2 - (3 - 2x^3)^2 = 2x(25x^2 - 30x + 9) - (9 - 12x^3 + 4x^6)$$

$$= -4x^6 + 62x^3 - 60x^2 + 18x - 9$$

$$(j) (3x - 5y)^2 - 16(x + 3y)^2 = 9x^2 - 30xy + 25y^2 - 16(x^2 + 6xy + 9y^2)$$

$$= -7x^2 - 126xy - 119y^2$$

3. Factoriser :

$$(a) 2ax - 4bx = 2x(a - 2b)$$

$$(b) x^3 + 3x^2 - 5x = x(x^2 + 3x - 5)$$

$$(c) 2a^2x - 3ax^2 + ax = ax(2a - 3x + 1)$$

$$(d) 6x^5y^3 - 24x^3y^5 + 3x^2y^8 - 15x^4y^3 = 3x^2y^3(2x^3 - 5x^2 - 8xy^2 + y^5)$$

$$(e) a(x + y) + 4(x + y) = (x + y)(a + 4)$$

$$(f) (a + b)(x - 2y) - 2b(x - 2y) = (x - 2y)(a + b - 2b)$$

$$= (x - 2y)(a - b)$$

$$(g) 2ax - x + 2a - 1 = x(2a - 1) + (2a - 1)$$

$$= (2a - 1)(x + 1)$$

$$(h) 3x + ax - 3y - ay = x(3 + a) - y(3 + a)$$

$$= (3 + a)(x - y)$$

$$(i) (x - 2y)(a + 2b) + (2a + b)(2y + x) = (x - 2y)(a + 2b - 2a - b) \\ = (x - 2y)(b - a)$$

$$(j) 2a^4 - a^3 + 2a^2 - a = a^3(2a - 1) + a(2a - 1) \\ = a(2a - 1)(a^2 + 1)$$

$$(k) x^3 + a^2x - ax^2 - a^3 = x(x^2 + a^2) - a(x^2 + a^2) \\ = (x^2 + a^2)(x - a)$$

$$(l) (x - y)(3a - 2b) + (y + x)(2a - 3b) - (x - y) = (x - y)(3a - 2b + 2a - 3b - 1) \\ = (x - y)(5a - 5b - 1)$$

$$(m) x^2 - 36 = (x - 6)(x + 6)$$

$$(n) 4x^2 - 4x + 1 = (2x - 1)^2$$

$$(o) 16x^2 + 1 + 8x = (4x + 1)^2$$

$$(p) -x^2 - y^2 + 2xy = -(x - y)^2$$

$$(q) 8x^2 - 24xy + 18y^2 = 2(4x^2 - 12xy + 9y^2) \\ = 2(2x - 3y)^2$$

4. Factoriser :

(a) $x^2 + 11x + 10 =$

div 10 : $\{ \pm 1, \pm 2, \pm 5, \pm 10 \}$
 $P(-1) = 0$

	1	11	10
-1		-1	-10
	1	10	0

$$x^2 + 11x + 10 = (x + 1)(x + 10)$$

(b) $xy^2 - xz^2 = x(y^2 - z^2) = x(y - z)(y + z)$

(c) $3ab^2 - 12ac^2 = 3a(b^2 - 4c^2) = 3a(b - 2c)(b + 2c)$

(d) $x^2 + 2x - 8 =$

div 8 = $\{ \pm 1, \pm 2, \pm 4, \pm 8 \}$

	1	2	-8
2		2	8
	1	4	0

$P(2) = 0$

$$x^2 + 2x - 8 = (x - 2)(x + 4)$$

$$(e) (a+b)^2 - c^2 = (a+b-c)(a+b+c)$$

$$(f) 2x^2 + 36x + 160 = 2(x^2 + 18x + 80)$$

$$\begin{array}{c|cc|c} & 1 & 18 & 80 \\ \hline -8 & & -8 & -8 \\ \hline & 1 & 10 & 0 \end{array}$$

die 80 $\left\{ \begin{array}{l} \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 16, \pm 20, \pm 40, \\ \pm 80 \end{array} \right.$
 $= 80$

$P(-8) = 0$

$$2x^2 + 36x + 160 = 2(x+8)(x+10)$$

$$(g) 1 - x^2 + 2xy - y^2 = 1 - (x^2 - 2xy + y^2) = 1 - (x-y)^2$$

$$= (1-x+y)(1+x-y)$$

$$(h) \frac{4}{25}a^2b^2 - \frac{1}{16}a^4c^2 = a^2 \left[\frac{4}{25}b^2 - \frac{1}{16}a^2c^2 \right]$$

$$= a^2 \left(\frac{2}{5}b - \frac{ac}{4} \right) \left(\frac{2}{5}b + \frac{ac}{4} \right)$$

$$(i) x^5 - x = x(x^4 - 1) = x(x^2 - 1)(x^2 + 1)$$

$$= x(x-1)(x+1)(x^2 + 1)$$

$$(j) \frac{3}{4}a^4 - \frac{4}{3}b^2 = \frac{9a^4 - 16b^2}{12} = \frac{1}{12} (3a^2 - 4b)(3a^2 + 4b)$$

$$(k) x^2 - 8x + 15 =$$

$$\text{Homen } (x-3)(x-5)$$

$$(l) (a^2 - 4a + 4) - (2a + 3)^2 = (a-2)^2 - (2a+3)^2$$

$$= (a-2-2a-3)(a-2+2a+3)$$

$$= (-a-5)(3a+1) = -(a+5)(3a+1)$$

$$(m) 18x^6y^4 + 2x^2y^6 - 12x^4y^5 =$$

$$2x^2y^4(9x^4 + y^2 - 6x^2y) = 2x^2y^4(3x^2 - y)^2$$

$$(n) a^4 - 2a^3b + 2ab^3 - b^4 = (a^4 - b^4) - 2ab(a^2 - b^2)$$

$$= (a-b)(a+b)(a^2+b^2) - 2ab(a-b)(a+b)$$

$$= (a-b)(a+b)(a^2 - ab + b^2) = (a-b)^3(a+b)$$

$$(o) x^4 - 3x^2 - 40 =$$

$$\text{Homen } \Rightarrow x^4 - 3x^2 - 40 = (x^2 + 5)(x^2 - 8)$$

$$= (x^2 + 5)(x - 2\sqrt{2})(x + 2\sqrt{2})$$

$$(p) 3x^2 - x - 2 =$$

$$\text{How } (x-1)(3x+2)$$

$$(q) 2x^3 + 7x^2 + 2x - 3 =$$

$$\text{How } (x+1)(x+3)(2x-1)$$

$$(r) 2a^2 + 3ab + b^2 =$$

$$\text{div } b^2 = \{ \pm 1, \pm b, \pm b^2 \}$$

$$P(-b) = 0$$

	2	3b	b ²
-b		-2b	-b ²
	2	b	0

$$2a^2 + 3ab + b^2 = (a + b)(2a + b)$$

Req : polynôme d'inconnue 'a'

5. Simplifier les fractions suivantes et poser les conditions d'existence :

$$(a) \frac{6ax}{3bx} = 2 \frac{a}{b}$$

$$\text{CE} : b \neq 0, x \neq 0$$

$$(b) \frac{3x^3y}{9xy^2} = \frac{x^2}{3y}$$

$$\text{CE} : x \neq 0, y \neq 0$$

$$(c) \frac{3(a+b)}{2(a+b)} = \frac{3}{2}$$

$$\text{CE} : a \neq -b$$

$$(d) \frac{-5a(b-c)}{15b(c-b)} = \frac{a}{3b}$$

CE: $b \neq c$

$$(e) \frac{5x-5a}{3x-3a} = \frac{5}{3}$$

CE: $x \neq a$

$$(f) \frac{3x^2-xy}{6xy-2y^2} = \frac{x}{2y}$$

CE: $y \neq 0, x \neq y$

$$(g) \frac{x^2-16}{x+4} = x-4$$

CE: $x \neq -4$

$$(h) \frac{9-x^2}{x^2-6x+9} = \frac{(3-x)(3+x)}{(x+3)^2}$$

CE $x \neq 3$

$$= \frac{\cancel{(3-x)}(3+x)}{(3-x)^2}$$
$$= \frac{3+x}{3-x}$$

$$(i) \frac{x^2 + x - 6}{2x(x+1) - 12} = \frac{x^2 + x - 6}{2x^2 + 2x - 12}$$

$$= \frac{\cancel{x^2 + x - 6}}{2(\cancel{x^2 + x - 6})} = \frac{1}{2}$$

CE: $x^2 + x - 6 \neq 0$
 $\Rightarrow x \neq -3, x \neq 2$

$$(j) \frac{(x^3 - x^2) - 4x + 4}{x^2 - 4} =$$

CE: $x \neq \pm 2$

$$= \frac{x^2(x-1) - 4(x-1)}{x^2 - 4} = \frac{\cancel{(x^2 - 4)}(x-1)}{\cancel{(x^2 - 4)}} = x-1$$

$$(k) \frac{x^2 + 9x + 14}{x^2 - 4} =$$

CE: $x \neq \pm 2$

$$\stackrel{(h)}{=} \frac{\cancel{(x+2)}(x+7)}{(x-2)\cancel{(x+2)}} = \frac{x+7}{x-2}$$

$$(l) \frac{9x - x^3}{x^3 + x^2 - 6x} = \frac{x(9 - x^2)}{x(x^2 + x - 6)}$$

$$= \frac{\cancel{x}(3-x)(3+x)}{\cancel{x}(x+3)(x-2)}$$

$$= \frac{3-x}{x-2}$$

CE: $x \neq 0$
 $x \neq -3, x \neq 2$

$$(m) \frac{(4x+5)^2}{8x^2+6x-5} =$$

$$= \frac{(4x+5)^2}{(2x-1)(4x+5)} = \frac{4x+5}{2x-1}$$

CE: $x \neq \frac{1}{2}, x \neq -\frac{5}{4}$

$$(n) \frac{x^3 - x^2 + 2}{x + 1} =$$

$$\underline{CE}: n \neq -1$$

$$\textcircled{H} = \frac{\cancel{n+1}(n^2 - 2n + 2)}{\cancel{n+1}}$$

$$= n^2 - 2n + 2$$

$$(o) \frac{x^2 + 2x - 3}{1 - x} =$$

$$\underline{CE}: n \neq 1$$

$$\textcircled{H} = \frac{\cancel{n-1}(n+3)}{\cancel{1-x}}$$

$$= -(n+3)$$

6. Effectuer, simplifier et poser les conditions d'existence :

$$(a) \frac{2x+1}{3x} + \frac{1-x}{3x} =$$

CE : $x \neq 0$

$$= \frac{2x+1+1-x}{3x}$$

$$= \frac{x+2}{3x}$$

$$(b) \frac{x-1}{3x^2} + \frac{8}{15x} =$$

CE : $x \neq 0$

$$= \frac{(x-1)5 + 8x}{3 \cdot 5 \cdot x^2}$$

$$= \frac{13x-5}{15x^2}$$

$$(c) \frac{2x+1}{4x} - \frac{x+3}{6x^2} =$$

CE : $x \neq 0$

$$= \frac{(2x+1)3x - (x+3)2}{2 \cdot 2 \cdot 3 \cdot x^2}$$

$$= \frac{6x^2 + 3x - 2x - 6}{12x^2}$$

$$= \frac{6x^2 + x - 6}{12x^2}$$

$$(d) \frac{4x-9}{3x} - \frac{3x+8}{4x} =$$

CE: $x \neq 0$

$$\begin{aligned} & \frac{4(4x-9) - 3(3x+8)}{3 \cdot 4 \cdot x} \\ &= \frac{16x - 36 - 9x - 24}{12x} \\ &= \frac{7x - 60}{12x} \end{aligned}$$

$$(e) \frac{2}{x-2} - \frac{5}{x+2} =$$

CE: $x \neq \pm 2$

$$\begin{aligned} & \frac{2(x+2) - 5(x-2)}{(x-2)(x+2)} \\ &= \frac{2x+4 - 5x+10}{(x-2)(x+2)} \\ &= \frac{-3x+14}{(x-2)(x+2)} \end{aligned}$$

$$(f) \frac{2}{x-2} - \frac{5}{x+2} =$$

$$\begin{aligned} & \frac{x-3}{x+3} + \frac{x+3}{x-3} \\ &= \frac{(x-3)^2 + (x+3)^2}{(x+3)(x-3)} \\ &= \frac{x^2 - 6x + 9 + x^2 + 6x + 9}{(x+3)(x-3)} \\ &= \frac{2(x^2 + 9)}{(x+3)(x-3)} \end{aligned}$$

CE: $x \neq \pm 3$

$$(g) x+2 - \frac{x^2-4}{x+2} =$$

$$\underline{CE}: n \neq -2$$

$$= \frac{(n+2)^2 - (n^2-4)}{n+2}$$

$$= \frac{n^2 + 4n + 4 - n^2 + 4}{n+2}$$

$$= \frac{4n+8}{n+2} = \frac{4(n+2)}{n+2} = 4$$

$$(h) \frac{2a}{a-1} + \frac{a^2+2a}{1-a^2} =$$

$$\underline{CE}: a \neq \pm 1$$

$$= \frac{2a}{a-1} + \frac{a^2+2a}{(-1+a)(1+a)}$$

$$= \frac{2a(a+1) - (a^2+2a)}{(a-1)(a+1)}$$

$$= \frac{2a^2 + 2a - a^2 - 2a}{(a-1)(a+1)}$$

$$= \frac{a^2}{(a-1)(a+1)}$$

$$(i) 5 - \frac{1}{x} + \frac{2x-3}{x^2-x} =$$

$$\underline{CE}: n \neq 0, n \neq 1$$

$$= \frac{5n(n-1) - (n-1) + 2n-3}{n(n-1)}$$

$$= \frac{5n^2 - 5n - n + 1 + 2n - 3}{n(n-1)}$$

$$= \frac{5n^2 - 4n - 2}{n(n-1)}$$

$$(j) \frac{1}{a^2 - b^2} - \frac{1}{a^2 + ab} =$$

$$\underline{\text{CE}}: a \neq \pm b, a \neq 0$$

$$= \frac{a - (a - b)}{a(a - b)(a + b)}$$

$$= \frac{b}{a(a - b)(a + b)}$$

$$(k) \frac{x + 3}{x^2 - 6x + 9} + \frac{2x + 3}{3x^2 - 9x} =$$

$$\underline{\text{CE}}: x \neq 3, x \neq 0$$

$$= \frac{x + 3}{(x - 3)^2} + \frac{2x + 3}{3x(x - 3)}$$

$$= \frac{3x(x + 3) + (2x + 3)(x - 3)}{3x(x - 3)^2}$$

$$= \frac{3x^2 + 9x + 2x^2 - 3x - 9}{3x(x - 3)^2}$$

$$= \frac{5x^2 + 6x - 9}{3x(x - 3)^2}$$

$$(l) \frac{5}{x^2 - 25} - \frac{1}{x^2 + 10x + 25} =$$

$$\underline{\text{CE}}: x \neq \pm 5$$

$$= \frac{5}{(x - 5)(x + 5)} - \frac{1}{(x + 5)^2}$$

$$= \frac{5(x + 5) - (x - 5)}{(x - 5)(x + 5)^2}$$

$$= \frac{4x + 30}{(x - 5)(x + 5)^2} = \frac{2(2x + 15)}{(x - 5)(x + 5)^2}$$

$$\begin{aligned}
 \text{(m)} \quad & \frac{3x}{4x^2 - 4x + 1} - \frac{3x + 2}{(2x - 1)^3} = \quad \text{CE: } x \neq \frac{1}{2} \\
 & \frac{3x}{(2x-1)^2} - \frac{3x+2}{(2x-1)^3} \\
 & = \frac{3x(2x-1) - (3x+2)}{(2x-1)^3} = \frac{6x^2 - 3x - 3x - 2}{(2x-1)^3} \\
 & = \frac{6x^2 - 6x - 2}{(2x-1)^3} = 2 \cdot \frac{3x^2 - 3x - 1}{(2x-1)^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(n)} \quad & \frac{4y}{y^2 + 4y + 3} + \frac{2}{y + 1} = \quad \text{CE: } y \neq -1, y \neq -3 \\
 & \text{(H)} \\
 & = \frac{4y + 2(y+3)}{(y+3)(y+1)} \\
 & = \frac{6y + 6}{(y+3)(y+1)} = \frac{6(y+1)}{(y+3)(y+1)} \\
 & = \frac{6}{y+3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(o)} \quad & \frac{3x}{x^2 + 3x - 10} + \frac{5}{4 - 2x} = \quad \text{CE: } x \neq 2, x \neq 5 \\
 & \text{(H)} \\
 & = \frac{3x}{(x-2)(x-5)} + \frac{-5}{2(-2+x)} \\
 & = \frac{6x - 5(x-5)}{2(x-2)(x-5)} = \frac{x + 10}{2(x-2)(x-5)}
 \end{aligned}$$

$$(p) \frac{5}{x^2 - y^2} + \frac{3x}{x^3 + x^2y} =$$

CE: $x \neq \pm y, x \neq 0$

$$= \frac{5}{(x-y)(x+y)} + \frac{3x}{x^2(x+y)}$$

$$= \frac{5x + 3(x-y)}{x(x-y)(x+y)}$$

$$= \frac{8x - 3y}{x(x-y)(x+y)}$$

$$(q) \frac{2x+4}{x^2+4x+4} - \frac{2x-1}{x^2+5x+6} =$$

CE: $x \neq -2, x \neq -3$

$$\stackrel{(H)}{=} \frac{2(x+2)}{(x+2)^2} - \frac{2x-1}{(x+2)(x+3)}$$

$$= \frac{2(x+3) - (2x-1)}{(x+2)(x+3)}$$

$$= \frac{7}{(x+2)(x+3)}$$

$$(r) \frac{42x}{x^2-49} + \frac{3x}{2x+14} - \frac{5-x}{7-x} =$$

CE: $x \neq \pm 7$

$$\frac{42x}{(x+7)(x-7)} + \frac{3x}{2(x+7)} - \frac{5-x}{7-x}$$

$$= \frac{84x + 3x(x-7) + 2(5-x)(x+7)}{2(x-7)(x+7)}$$

$$= \frac{84x + 3x^2 - 21x + 2(-x^2 - 2x + 35)}{2(x-7)(x+7)}$$

$$\stackrel{D}{=} \frac{x^2 + 59x + 70}{2(x-7)(x+7)}$$

$$(s) \frac{x^3 - 36x}{2x^2 + 12x} + \frac{x^2 + 11x + 30}{4x^2 - 100} =$$

CE: $x \neq 0, x \neq \pm 5, x \neq -6$

$$\textcircled{1} = \frac{x(x^2 - 36)}{2x(x+6)} + \frac{\cancel{(x+5)}(x+6)}{4(x-5)\cancel{(x+5)}}$$

$$= \frac{\cancel{x}(x-6)\cancel{(x+6)}}{2\cancel{x}\cancel{(x+6)}} + \frac{x+6}{4(x-5)}$$

$$= \frac{2(x-6)(x-5) + (x+6)}{4(x-5)}$$

$$= \frac{2(x^2 - 11x + 30) + x + 6}{4(x-5)}$$

$$= \frac{2x^2 - 21x + 66}{4(x-5)}$$

$$(t) \frac{3x}{x^2 - 9x + 14} - \frac{2x}{x^2 + x - 6} + \frac{x}{x^2 - 4x - 21} =$$

$\textcircled{1}$
CE: $x \neq 2, x \neq 7$
 $x \neq -3$

$$\textcircled{1} = \frac{3x}{(x-2)(x-7)} - \frac{2x}{(x-2)(x+3)} + \frac{x}{(x+3)(x-7)}$$

$$= \frac{3x(x+3) - 2x(x-7) + x(x-2)}{(x-2)(x+3)(x-7)}$$

$$= \frac{3x^2 + 9x - 2x^2 + 14x + x^2 - 2x}{(x-2)(x+3)(x-7)}$$

$$= \frac{2x^2 + 21x}{(x-2)(x+3)(x-7)}$$

$$= \frac{x(2x + 21)}{(x-2)(x+3)(x-7)}$$

7. Résoudre les équations suivantes :

(a) $3x - 8 = 7x + 6$

$$\Leftrightarrow -4x = 14 \quad \Leftrightarrow x = -\frac{7}{2}$$

$$S: \left\{ -\frac{7}{2} \right\}$$

(b) $5x + 3 - 6x = 4 - 8x - 2$

$$\Leftrightarrow 7x = -1 \quad \Leftrightarrow x = -\frac{1}{7}$$

$$S: \left\{ -\frac{1}{7} \right\}$$

(c) $\frac{1}{4}x = 12$

$$\Leftrightarrow x = 48$$

$$S: \{ 48 \}$$

$$(d) 9x + 3 = 7x - 2$$

$$\Leftrightarrow 2x = -5 \quad (\Leftrightarrow) \quad x = -\frac{5}{2}$$

$$S: \left\{ -\frac{5}{2} \right\}$$

$$(e) 4(x+1) + 3(2x-1) = 0$$

$$\Leftrightarrow 4x + 4 + 6x - 3 = 0 \quad (\Leftrightarrow) \quad 10x = -1$$

$$\Leftrightarrow x = -\frac{1}{10}$$

$$S: \left\{ -\frac{1}{10} \right\}$$

$$(f) 4x - (x+3) = 12 - 3(x-2)$$

$$\Leftrightarrow 4x - x - 3 = 12 - 3x + 6$$

$$\Leftrightarrow 6x = 21 \quad (\Leftrightarrow) \quad x = \frac{21}{6} \quad (\Leftrightarrow) \quad x = \frac{7}{2}$$

$$S: \left\{ \frac{7}{2} \right\}$$

$$(g) \frac{1}{4}x - \frac{2}{3} = \frac{5}{12}x$$

$$\Leftrightarrow \frac{3x - 8}{12} = \frac{5x}{12}$$

$$\Leftrightarrow -2x = 8$$

$$\Leftrightarrow x = -4$$

$$S: \{-4\}$$

$$(h) \frac{x}{3} + 3 = \frac{x}{5} - \frac{1}{2}$$

$$\Leftrightarrow \frac{10x + 90}{30} = \frac{6x - 15}{30}$$

$$\Leftrightarrow 4x = -105 \quad \Leftrightarrow x = -\frac{105}{4}$$

$$S: \left\{-\frac{105}{4}\right\}$$

$$(i) \frac{x+5}{7} = \frac{x}{4} + \frac{1}{2}$$

$$\Leftrightarrow \frac{4x + 20}{28} = \frac{7x + 14}{28}$$

$$\Leftrightarrow -3x = -6$$

$$\Leftrightarrow x = 2$$

$$S: \{2\}$$

$$(j) \frac{1}{3}(x-2) = \frac{1}{5}(x+4) + 2$$

$$\Leftrightarrow \frac{5(x-2)}{\cancel{15}} = \frac{3(x+4) + 30}{\cancel{15}}$$

$$\Leftrightarrow 5x - 10 = 3x + 12 + 30$$

$$\Leftrightarrow 2x = 52$$

$$\Leftrightarrow x = 26$$

$$S: \{ 26 \}$$

$$(k) \frac{x-7}{6} = -\frac{1}{2}$$

$$\Leftrightarrow \frac{x-7}{\cancel{6}} = \frac{-3}{\cancel{6}}$$

$$\Leftrightarrow x = 4$$

$$S: \{ 4 \}$$

$$(l) \frac{3x-7}{5} = \frac{4-3x}{3}$$

$$\Leftrightarrow \frac{3(3x-7)}{\cancel{15}} = \frac{5(4-3x)}{\cancel{15}}$$

$$\Leftrightarrow 9x - 21 = 20 - 15x$$

$$\Leftrightarrow 24x = 41$$

$$\Leftrightarrow x = \frac{41}{24}$$

$$S: \left\{ \frac{41}{24} \right\}$$

$$(m) \frac{x+3}{4} = 4x - 2(x-3)$$

$$\Leftrightarrow x+3 = 4[4x - 2x + 6]$$

$$\Leftrightarrow x+3 = 16x - 8x + 24$$

$$\Leftrightarrow -7x = 21$$

$$\Leftrightarrow x = -3$$

$$S: \{-3\}$$

$$(n) 3(x+3) = (2-x) + 2(2x+1)$$

$$\Leftrightarrow 3x+9 = 2-x+4x+2$$

$$\Leftrightarrow 0x = -5 \quad \text{imp.}$$

$$S: \emptyset$$

$$(o) \frac{3x}{2} - \frac{5x}{4} = 2 - \frac{3-x}{8}$$

$$\Leftrightarrow \frac{12x - 10x}{8} = \frac{16 - 3 + x}{8}$$

$$\Leftrightarrow x = 13$$

$$S: \{13\}$$

$$(p) 5x - 2 + (x - 2)(x - 3) = (x + 1)^2$$

$$\Leftrightarrow 5x - 2 + \cancel{x^2} - 2x - 3x + 6 = \cancel{x^2} + 2x + 1$$

$$\Leftrightarrow -2x = -3$$

$$\Leftrightarrow x = \frac{3}{2}$$

$$S = \left\{ \frac{3}{2} \right\}$$

$$(q) 3x - \frac{1}{2}(4 - x) = x - \frac{1}{3}$$

$$\Leftrightarrow \frac{18x - 3(4 - x)}{\cancel{6}} = \frac{6x - 2}{\cancel{6}}$$

$$\Leftrightarrow 18x - 12 + 3x = 6x - 2$$

$$\Leftrightarrow 15x = 10$$

$$\Leftrightarrow x = \frac{2}{3}$$

$$S = \left\{ \frac{2}{3} \right\}$$

$$(r) 5(2x - 1) = 2(3 + 5x) - 11$$

$$\Leftrightarrow 10x - 5 = 6 + 10x - 11$$

$$\Leftrightarrow 0x = 0$$

$$S = \mathbb{R}$$

$$(s) \frac{2x+2}{4} - \frac{2x+3}{2} = \frac{1}{6}$$

$$\Rightarrow \frac{3(2x+2) - 6(2x+3)}{\cancel{12}} = \frac{2}{\cancel{12}}$$

$$\Leftrightarrow 6x + 6 - 12x - 18 = 2$$

$$\Leftrightarrow -6x = 14$$

$$\Leftrightarrow x = -\frac{7}{3}$$

$$S: \left\{ -\frac{7}{3} \right\}$$

$$(t) \left(\frac{7}{2} - \frac{2x-3}{2} - (3x+5) = \frac{3}{2} - \frac{5x}{2} - x \right) \times 2$$

$$\Rightarrow 7 - (2x-3) - 2(3x+5) = 3 - 5x - 2x$$

$$\Rightarrow 7 - 2x + 3 - 6x - 10 = 3 - 7x$$

$$\Leftrightarrow -2x = 3$$

$$\Leftrightarrow x = -\frac{3}{2}$$

$$S: \left\{ -\frac{3}{2} \right\}$$

8. Résoudre les inéquations suivantes :

(a) $2x - 3 \geq -7$

$$\Leftrightarrow 2x \geq -4$$

$$\Leftrightarrow x \geq -2$$

$$S: [-2, +\infty)$$

(b) $4x + 3 \geq 5x - 2$

$$\Leftrightarrow -x \geq -5$$

$$\Leftrightarrow x \leq 5$$

$$S: (-\infty, 5]$$

(c) $\frac{3}{4}x - 6 \leq 5 + 2x$

$$\Leftrightarrow \frac{3}{4}x - 2x \leq 11$$

$$\Leftrightarrow -\frac{5x}{4} \leq 11$$

$$\Leftrightarrow x \geq -\frac{44}{5}$$

$$S: \left[-\frac{44}{5}, +\infty\right)$$

$$(d) 4 - 16x \leq 6 - 5(3x - 2)$$

$$\Leftrightarrow 4 - 16x \leq 6 - 15x + 10$$

$$\Leftrightarrow -x \leq 12$$

$$\Leftrightarrow x \geq -12$$

$$S: [-12, +\infty)$$

$$(e) 5(2x - 1) + 3(4 - x) < 0$$

$$\Leftrightarrow 10x - 5 + 12 - 3x < 0$$

$$\Leftrightarrow 7x < -7$$

$$\Leftrightarrow x < -1$$

$$S: -\infty, -1[$$

$$(f) 3(3x - 1) - 2(2x + 2) \leq 3 - 5x$$

$$\Leftrightarrow 9x - 3 - 4x - 4 \leq 3 - 5x$$

$$\Leftrightarrow 10x \leq 10$$

$$\Leftrightarrow x \leq 1$$

$$S: -\infty, 1]$$

$$(g) \frac{x}{2} + \frac{x}{3} + \frac{x}{4} + \frac{2}{5} > 3$$

$$\Leftrightarrow \frac{30n + 20n + 15n + 24}{60} > \frac{180}{60}$$

$$\Leftrightarrow 65n > 156$$

$$\Leftrightarrow n > \frac{156}{65}$$

$$\Leftrightarrow n > \frac{12}{5} \quad S: \left] \frac{12}{5}, +\infty \right[$$

$$(h) \frac{1}{2}(x+2) - 1 > \frac{x}{3} - 5 + x$$

$$\Leftrightarrow \frac{3(n+2) - 2}{6} > \frac{2n - 30 + 6n}{6}$$

$$\Leftrightarrow 3n + 6 - 2 > 8n - 30$$

$$\Leftrightarrow -5n > -36$$

$$\Leftrightarrow n < 6$$

$$S:]-\infty, 6[$$

$$(i) \frac{1+2x}{5} + \frac{4-x}{3} \geq \frac{7}{30}$$

$$\Leftrightarrow \frac{6(1+2n) + 10(4-n)}{30} \geq \frac{7}{30}$$

$$\Leftrightarrow 6 + 12n + 40 - 10n \geq 7$$

$$\Leftrightarrow 2n \geq -39$$

$$\Leftrightarrow n \geq -\frac{39}{2}$$

$$S: \left[-\frac{39}{2}, +\infty \right[$$

$$(j) \frac{3x-5}{4} - (3-2x) \geq \frac{2x-1}{3} + 4x$$

$$\Leftrightarrow \frac{3(3x-5) - 12(3-2x)}{12} \geq \frac{4(2x-1) + 48x}{12}$$

$$\Leftrightarrow 9x - 15 - 36 + 24x \geq 8x - 4 + 48x$$

$$\Leftrightarrow -23x \geq 47$$

$$\Leftrightarrow x \leq -\frac{47}{23}$$

$$S: -\infty, -\frac{47}{23}]$$

9. Résoudre les équations suivantes :

$$(a) (3x+1) \cdot (5x-2) = 0$$

$$\Leftrightarrow \begin{cases} 3x+1=0 \\ 5x-2=0 \end{cases} \Leftrightarrow \begin{cases} x = -\frac{1}{3} \\ x = \frac{2}{5} \end{cases}$$

$$S: \left\{ -\frac{1}{3}, \frac{2}{5} \right\}$$

$$(b) 3x^2 \cdot (4-3x) \cdot (2x+7) = 0$$

$$\Leftrightarrow \begin{cases} 3x^2=0 \\ 4-3x=0 \\ 2x+7=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ x=\frac{4}{3} \\ x=-\frac{7}{2} \end{cases}$$

$$S: \left\{ -\frac{7}{2}, 0, \frac{4}{3} \right\}$$

$$(c) 3x^2 + 4x = 0$$

$$\Leftrightarrow x(3x + 4) = 0$$

$$\Leftrightarrow \begin{cases} x = 0 \\ 3x + 4 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 0 \\ x = -\frac{4}{3} \end{cases}$$

$$S: \left\{ -\frac{4}{3}, 0 \right\}$$

$$(d) 2x^2 - 32 = 0$$

$$\Leftrightarrow 2(x^2 - 16) = 0 \Leftrightarrow 2(x - 4)(x + 4) = 0$$

$$\Leftrightarrow \begin{cases} x - 4 = 0 \\ x + 4 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 4 \\ x = -4 \end{cases}$$

$$S: \left\{ -4, 4 \right\}$$

$$(e) x^2 + 2x + 1 = 0$$

$$\Leftrightarrow (x + 1)^2 = 0 \Leftrightarrow x + 1 = 0 \Leftrightarrow x = -1$$

$$S: \left\{ -1 \right\}$$

$$(f) 4x^2 = \frac{1}{9}$$

$$\Leftrightarrow 4x^2 - \frac{1}{9} = 0 \quad \Leftrightarrow \left(2x - \frac{1}{3}\right)\left(2x + \frac{1}{3}\right) = 0$$

$$\Leftrightarrow \begin{cases} 2x - \frac{1}{3} = 0 \\ 2x + \frac{1}{3} = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = \frac{1}{6} \\ x = -\frac{1}{6} \end{cases}$$

$$S: \left\{ -\frac{1}{6}, \frac{1}{6} \right\}$$

$$(g) x(x-3) + 2(x-3) = 0$$

$$\Leftrightarrow (x-3)(x+2) = 0$$

$$\Leftrightarrow \begin{cases} x+2 = 0 \\ x-3 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = -2 \\ x = 3 \end{cases}$$

$$S: \{-2, 3\}$$

$$(h) 2x(x+1) = 5(x+1)$$

$$\Leftrightarrow 2x(x+1) - 5(x+1) = 0$$

$$\Leftrightarrow (x+1)(2x-5) = 0$$

$$\Leftrightarrow \begin{cases} x+1 = 0 \\ 2x-5 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = -1 \\ x = \frac{5}{2} \end{cases}$$

$$S: \left\{ -1, \frac{5}{2} \right\}$$

$$(i) 12x^3 - 27x = 0$$

$$\Leftrightarrow 3x(4x^2 - 9) = 0 \Leftrightarrow 3x(2x - 3)(2x + 3) = 0$$

$$\Leftrightarrow \begin{cases} 3x = 0 \\ 2x - 3 = 0 \\ 2x + 3 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = \frac{3}{2} \\ x = -\frac{3}{2} \end{cases}$$

$$S: \left\{ -\frac{3}{2}, 0, \frac{3}{2} \right\}$$

$$(j) x^2 - 36 = 2x + 12$$

$$\Leftrightarrow (x^2 - 36) - (2x + 12) = 0 \quad \text{ou développer}$$

$$\Leftrightarrow (x - 6)(x + 6) - 2(x + 6) = 0 \quad + \text{Mener}$$

$$\Leftrightarrow (x + 6)(x - 6 - 2) = 0$$

$$\Leftrightarrow (x + 6)(x - 8) = 0$$

$$\Leftrightarrow \begin{cases} x + 6 = 0 \\ x - 8 = 0 \end{cases} \Leftrightarrow \begin{cases} x = -6 \\ x = 8 \end{cases}$$

$$S: \{-6, 8\}$$

$$(k) (3x - 1)^2 - 4x^2 = 0$$

$$\Leftrightarrow [(3x - 1) - 2x][(3x - 1) + 2x] = 0$$

$$\Leftrightarrow (x - 1)(5x - 1) = 0$$

$$\Leftrightarrow \begin{cases} x - 1 = 0 \\ 5x - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ x = \frac{1}{5} \end{cases}$$

$$S: \left\{ \frac{1}{5}, 1 \right\}$$

$$(l) 4x^2 + 1 = 4x$$

$$\Leftrightarrow 4x^2 - 4x + 1 = 0 \quad \Leftrightarrow (2x - 1)^2 = 0$$

$$\Leftrightarrow 2x - 1 = 0 \quad \Leftrightarrow x = \frac{1}{2}$$

$$S: \left\{ \frac{1}{2} \right\}$$

$$(m) (x^2 - 9)^2 = (x + 3)^2$$

$$\Leftrightarrow (x^2 - 9)^2 - (x + 3)^2 = 0$$

$$\Leftrightarrow [(x^2 - 9) - (x + 3)][(x^2 - 9) + (x + 3)] = 0$$

$$\Leftrightarrow [(x - 3)(x + 3) - (x + 3)][(x - 3)(x + 3) + (x + 3)] = 0$$

$$\Leftrightarrow (x + 3)[(x - 3) - 1] \cdot (x + 3)[(x - 3) + 1] = 0$$

$$\Leftrightarrow (x + 3)^2 (x - 4)(x - 2) = 0$$

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$$(n) (2x - 1)^2 = (2x - 1)(x + 3)$$

$$\Leftrightarrow (2x - 1)^2 - (2x - 1)(x + 3) = 0$$

$$\Leftrightarrow (2x - 1)[(2x - 1) - (x + 3)] = 0$$

$$\Leftrightarrow (2x - 1)(x - 4) = 0$$

$$\Leftrightarrow \begin{cases} 2x - 1 = 0 \\ x - 4 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{2} \\ x = 4 \end{cases}$$

$$S: \left\{ \frac{1}{2}, 4 \right\}$$

$$(*) \Leftrightarrow \begin{cases} (x+3)^2 = 0 \\ x-4 = 0 \\ x-2 = 0 \end{cases} \Leftrightarrow \begin{cases} x+3 = 0 \\ x = 4 \\ x = 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -3 \\ x = 4 \\ x = 2 \end{cases}$$

$$S: \{-3, 2, 4\}$$

(H) = Horner

(o) $x^2 - 8x - 20 = 0$

\rightarrow div-bin : $P(-2) = 0$

	1	-8	-20
-2		-2	20
	1	-10	0

$\Rightarrow (x + 2)(x - 10) = 0$

$\Leftrightarrow \begin{cases} x + 2 = 0 \\ x - 10 = 0 \end{cases} \Leftrightarrow \begin{cases} x = -2 \\ x = 10 \end{cases}$

$S: \{-2, 10\}$

(p) $x^2 - 15x + 26 = 0$

\rightarrow div-bin : $P(2) = 0$

	1	-15	26
2		2	-26
	1	-13	0

$\Rightarrow (x - 2)(x - 13) = 0$

$\Leftrightarrow \begin{cases} x - 2 = 0 \\ x - 13 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ x = 13 \end{cases}$

$S: \{2, 13\}$

(q) $2x^3 - 8x^2 + 6x = 0$

$\Leftrightarrow 2x(x^2 - 4x + 3) = 0$

(H) $\Leftrightarrow 2x(x - 3)(x - 1) = 0$ (cf eno pedt)

$\Leftrightarrow \begin{cases} 2x = 0 \\ x - 3 = 0 \\ x - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 3 \\ x = 1 \end{cases}$

$S: \{0, 1, 3\}$

$$(r) 2x^3 + 3x^2 = 8x + 12$$

$$\Leftrightarrow 2x^3 + 3x^2 - 8x - 12 = 0$$

$$\Leftrightarrow x^2(2x+3) - 4(2x+3) = 0$$

$$\Leftrightarrow (2x+3)(x^2-4) = 0$$

$$\Leftrightarrow (2x+3)(x-2)(x+2) = 0$$

$$\Leftrightarrow \begin{cases} 2x+3=0 \\ x-2=0 \\ x+2=0 \end{cases} \Leftrightarrow \begin{cases} x = -\frac{3}{2} \\ x = 2 \\ x = -2 \end{cases} \quad S: \left\{ -2, -\frac{3}{2}, 2 \right\}$$

$$(s) 2x^5 + x^4 - 10x^3 = 0$$

$$\Leftrightarrow x^3(2x^2 + x - 10) = 0$$

$$\Leftrightarrow x^3(2x+5)(x-2) = 0$$

$$\Leftrightarrow \begin{cases} x^3=0 \\ 2x+5=0 \\ x-2=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ x=-\frac{5}{2} \\ x=2 \end{cases}$$

$$S: \left\{ -\frac{5}{2}, 0, 2 \right\}$$

$$(t) 3x^3 + 8x^2 - 15x + 4 = 0$$

$$\Leftrightarrow (x-1)(x+4)(3x-1) = 0$$

$$\Leftrightarrow \begin{cases} x-1=0 \\ x+4=0 \\ 3x-1=0 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ x=-4 \\ x=\frac{1}{3} \end{cases}$$

$$S: \left\{ -4, \frac{1}{3}, 1 \right\}$$